# Ice dynamics and physics in a next-generation ice sheet model

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# Outline

***introduction***

conservation of momentum

conservation of energy

conservation of mass

boundary conditions

ice physics

physical processes

other

# Goal of this talk

Give context for breakout session on ***ice dynamics and physics***[1] by suggesting areas where we are doing:

[1]to be defined … [2]3:00 pm, Jemez room

## Goal of *ice dynamics* breakout session

provide list of dynamics & physics to be included in the ideal ice sheet model & prioritize by:

# Terminology

*Dynamics* = equations of motion

*Physics* = everything else

What do we mean by "***good enough***"?

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# Equations of motion

**(1) Full Stokes[1] (*u, v, w, P*)** ***… L >> H …***

$$\hat{x}: \quad \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial P}{\partial x} = 0, \qquad \hat{z}: \quad \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial x} - \frac{\partial P}{\partial z} = \rho g$$

[1]ELMER [2]Blatter/Pattyn [3]GLIMMER

# Equations of motion (cont…)

**(4)** Depth integrated 1$^{st}$-order - SSA; shelf w/ basal traction

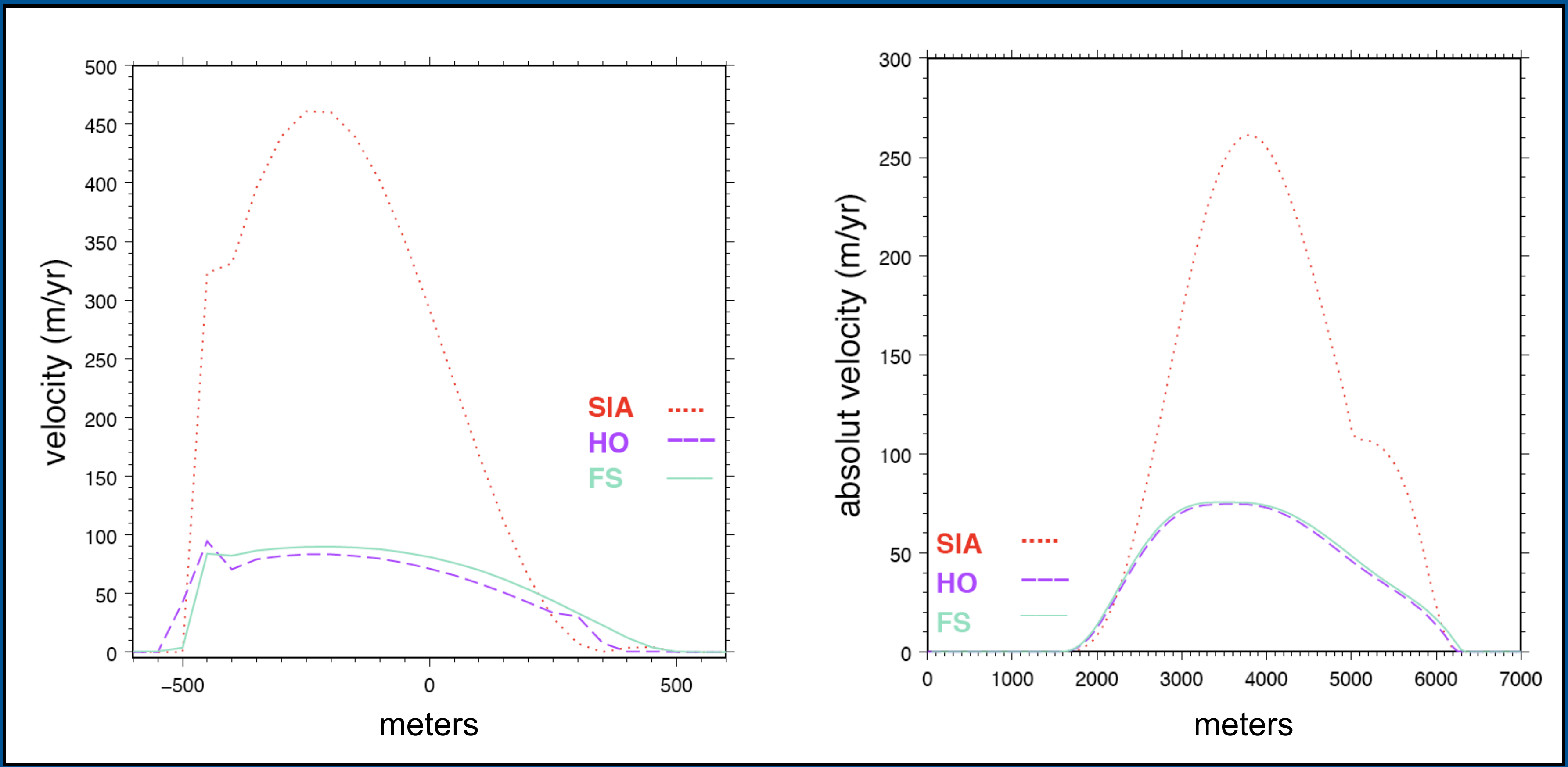
**(5)** Hybrid in *horizontal* - (3) and (4) - SSA "glued" to SIA

**(6)** Hybrid in *vertical* - (3) and (4) - SIA w/ sliding via SSA[1]

**(7)** Other "higher-order" schemes (?)

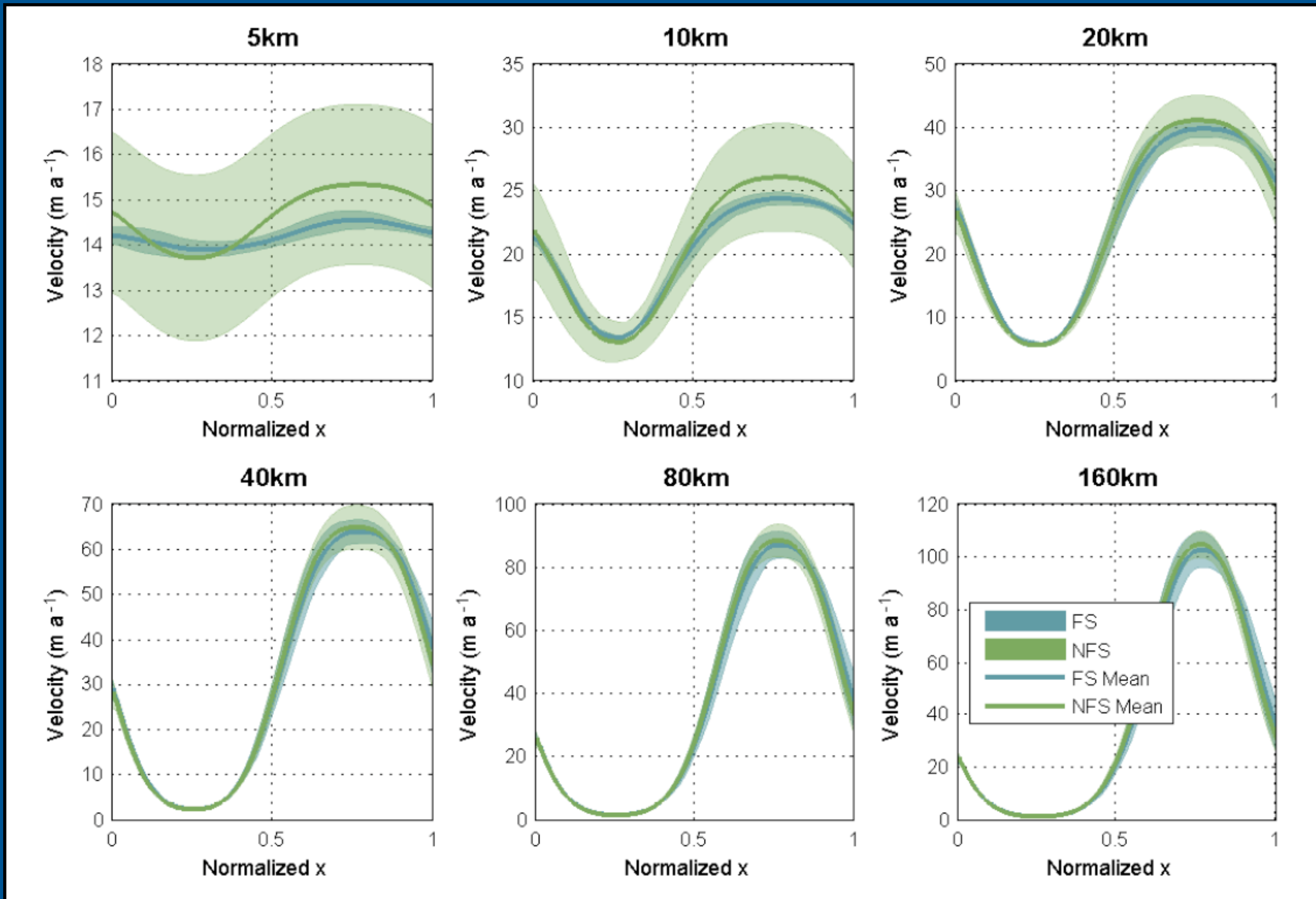
[1]Bueler and Brown (submitted)

# 0-, 1$^{st}$-order SIA vs. FS

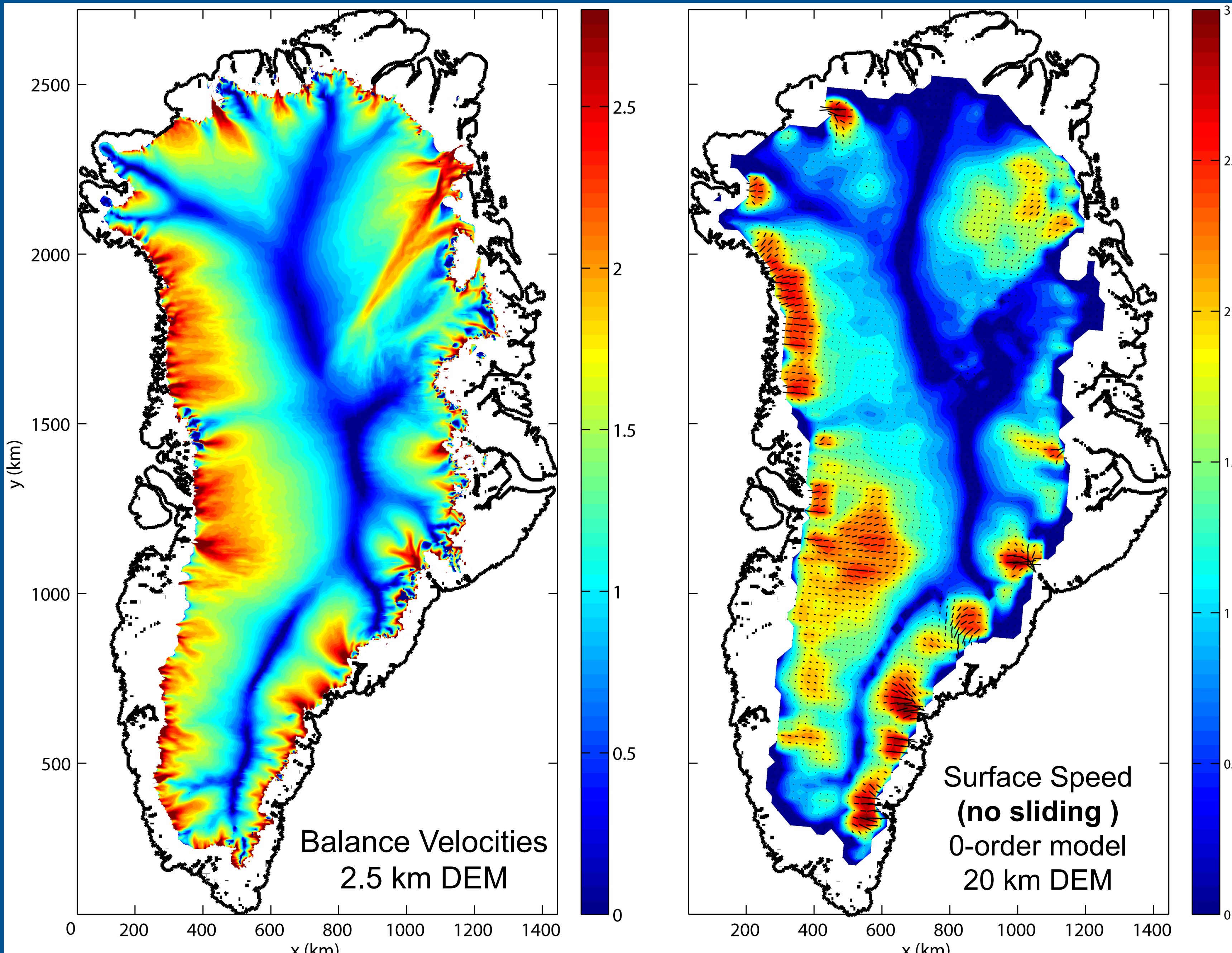



Schafer et al. (*TCD*)

# 1st -order SIA vs. FS (no slip)

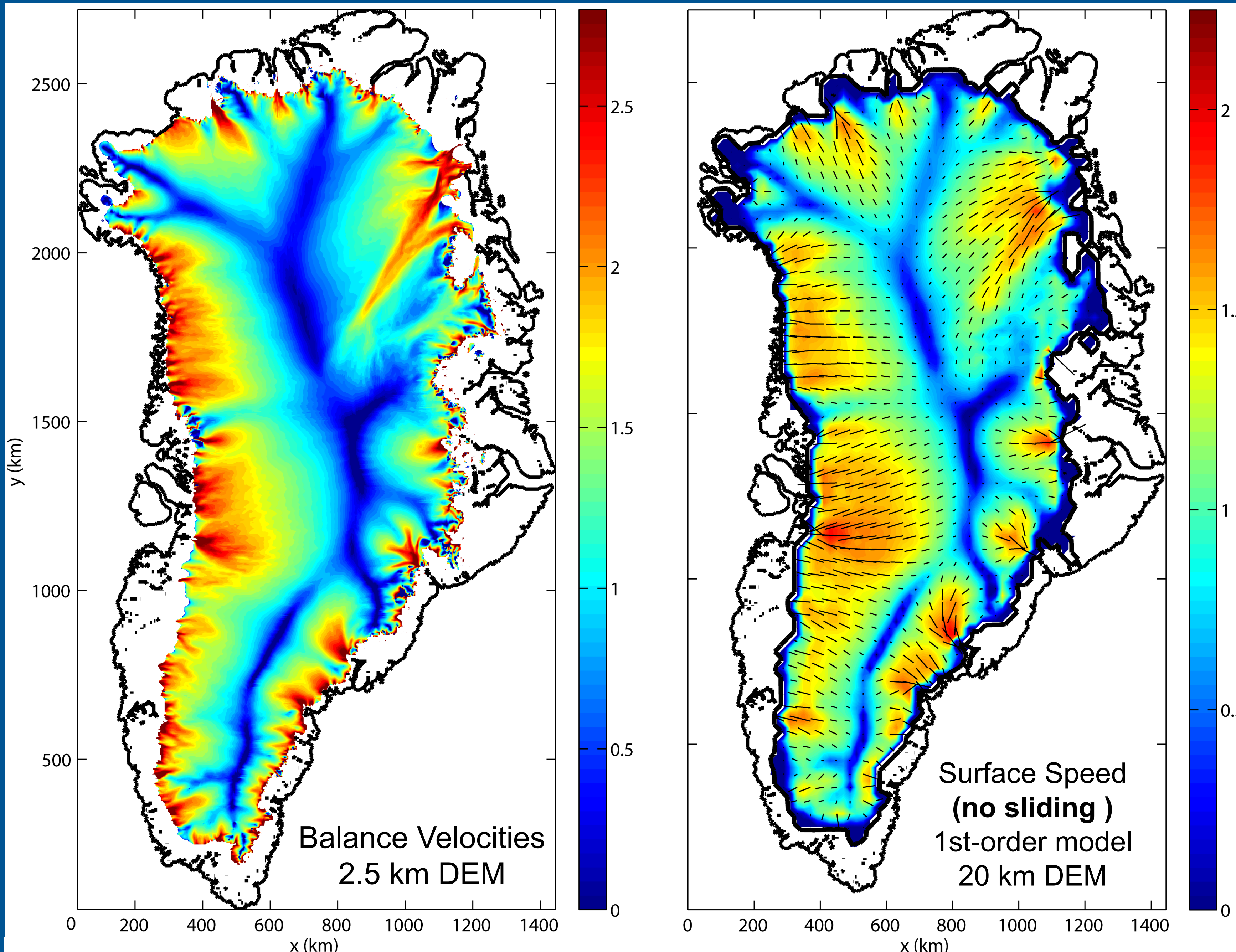



Pattyn et al. (*TC*, 2008)

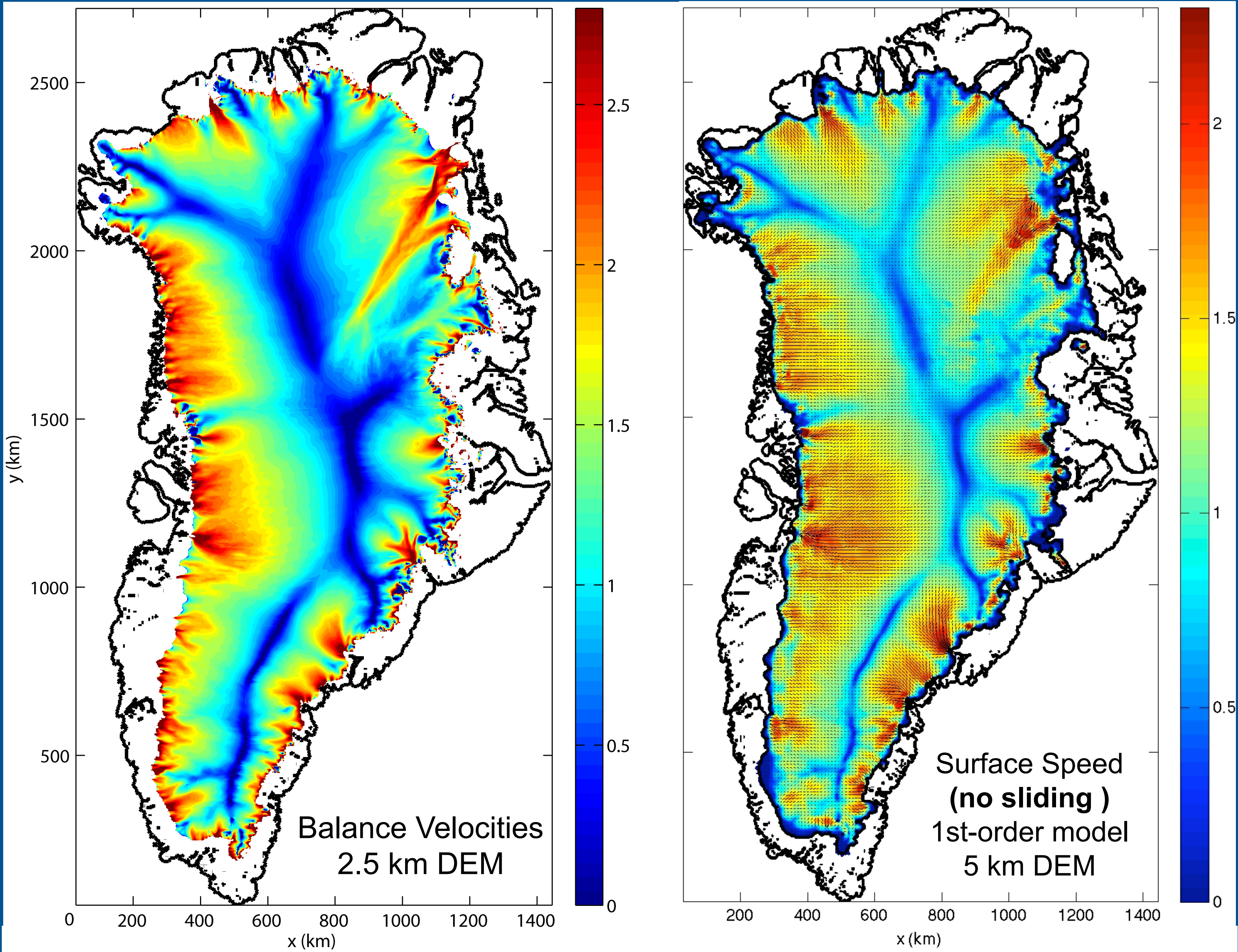



Bamber et al. (*J.Glac.*, v.46, 2000)

Bamber et al. (*Ann.Glac.*, v.30, 2000)




Bamber et al. (*J.Glac.*, v.46, 2000)




(Bamber et al. (*J.Glac.*, v.46, 2000)

## CPU time[1] (approx.)

| | SIA | HO | FS |
|---|---|---|---|
| **Diagnostic** | 1 (0.06 sec) | $\sim 10^2$ | $\sim 10^4$ |
| **Prognostic** | 1 (0.3 sec) | $\sim 10^3$ | $\sim 10^4$ |

[1]Schafer et al. (*TCD*)

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# Conservation of energy

We can solve either the …

(1) heat equation, or

(2) enthalpy equation (?)

# Conservation of energy

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot \left( k \nabla T \right) - \rho c \mathbf{u} \cdot \nabla T + \boldsymbol{\sigma}_{ij} \dot{\varepsilon}_{ij}$$

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# Conservation of mass

$$(1) \quad \frac{\partial H}{\partial t} = -\nabla \cdot \left(\overline{\mathbf{u}} H\right) + \dot{b} - \dot{m}$$

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# Dynamic Boundary Conditions

**surface:** continuous traction

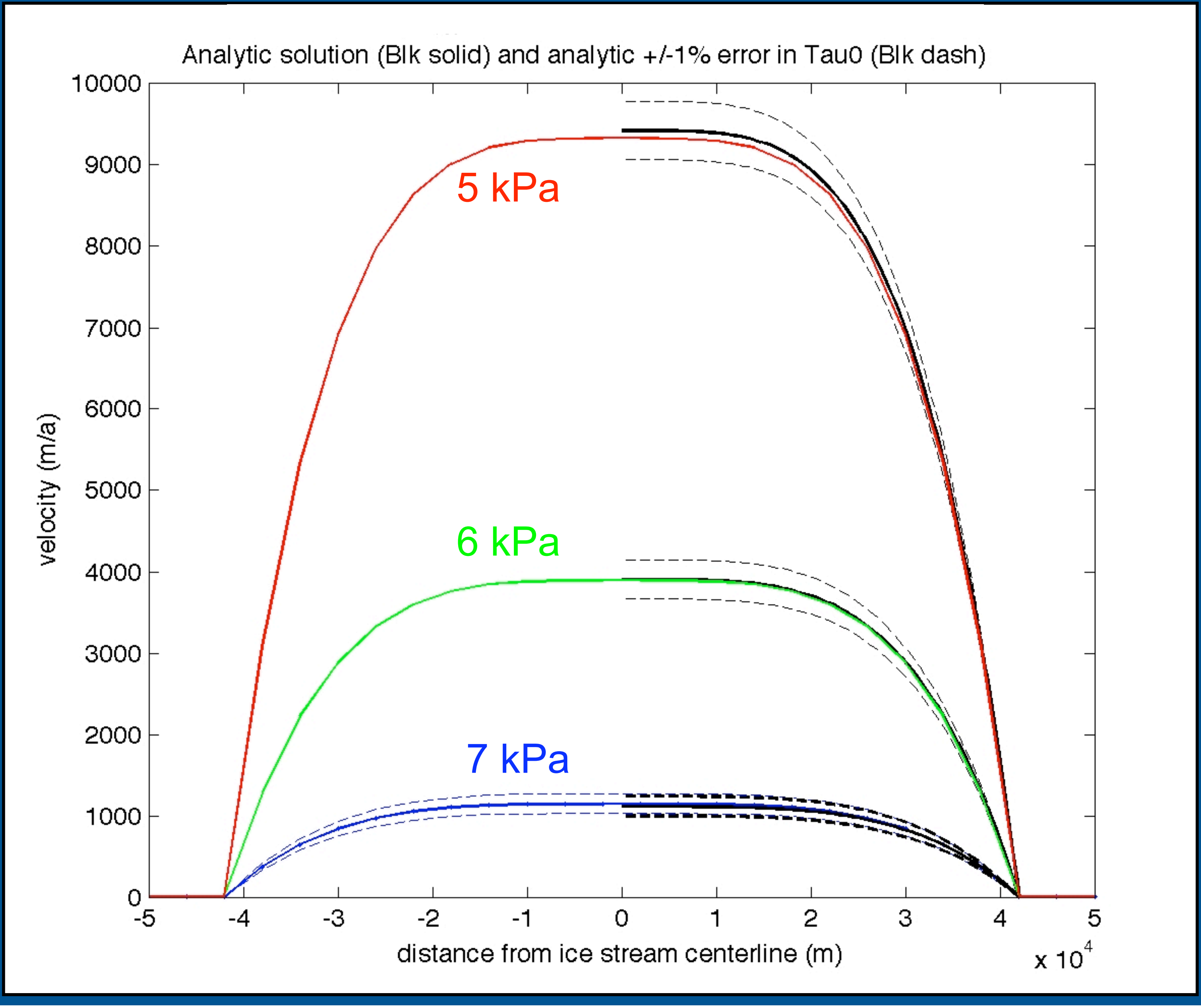
$$\left(\tau_{ij} - P\delta_{ij}\right)n_j = -P_{atm}n_i = 0$$

# Dynamic Boundary Conditions (bed)

Note that all three basal bcs can be captured by $B^2$ type sliding law:

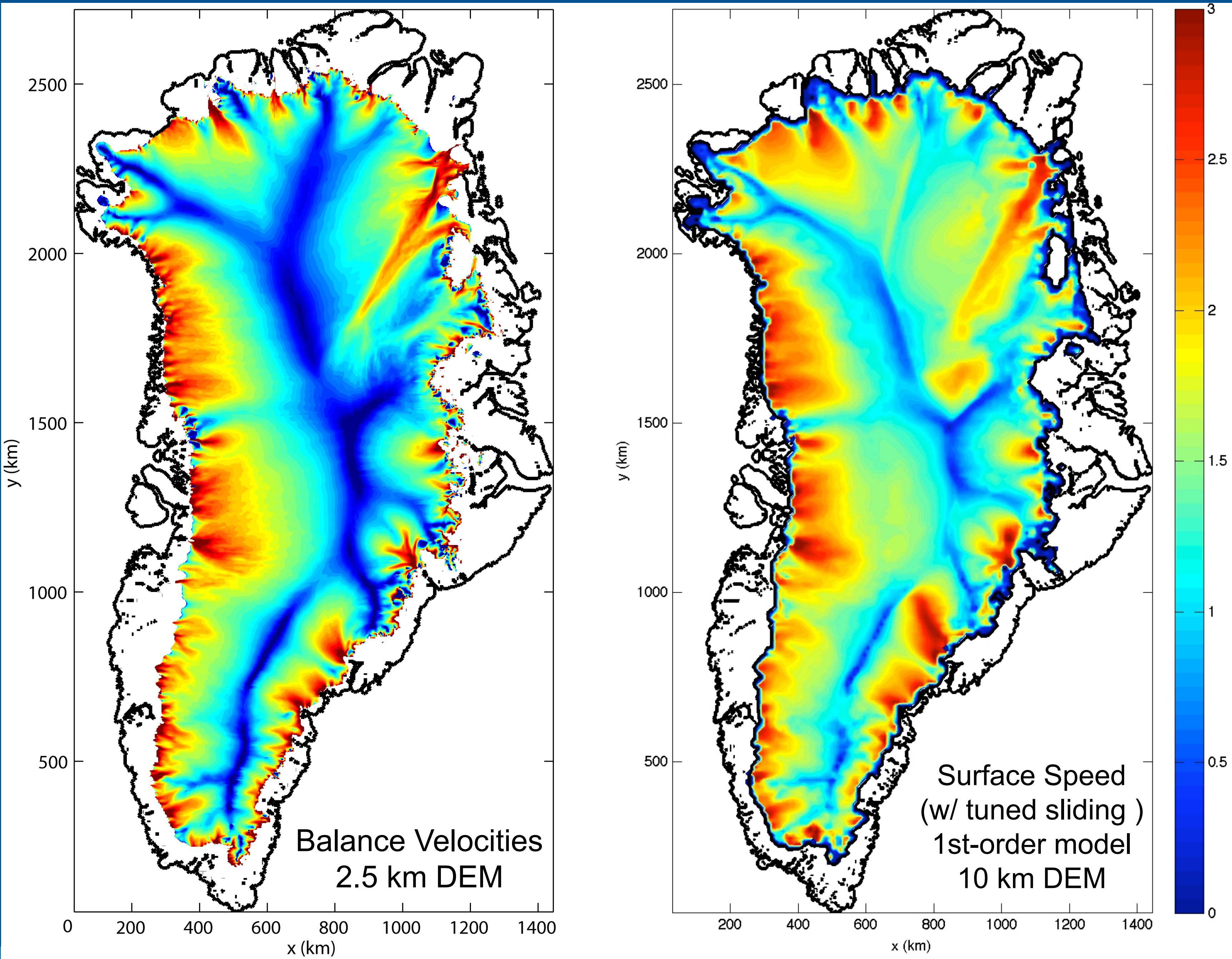
$$\tau_b = B^2 u$$

# Sliding with specified basal yield stress (1st order model)



# Dynamic Boundary Conditions (bed)

Assuming $B^2$ sliding law, where does $B^2$ (or [illegible]$_0$) come from?




Bamber et al. (*J.Glac.*, v.46, 2000)

# Temperature Boundary Conditions

# Thickness Boundary Conditions

?

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# Ice Physics

(1) isotropic flow law

(2) anisotropic flow law

(3) special rheology (e.g. basal ice, shear margins)

# Ice Physics

**(1) isotropic flow law**

(i) is Glenn's law w/ $n$=3 "good enough"?

(ii) is Glenn's law w/ possibility of $1 \leq n \leq 4$ needed[1] ?

(iii) $n$=3 (normal), $n$=1 (low stress/strain) needed[2] ?

field sites or not at all.

[1]Goldsby & Kohlstedt (2001), [2]Pettit (2003)

# Ice Physics

## (2) anisotropic flow law

Simplified (2d) methods[1] of accounting for anisotropy calculate scalar *E(x,z)* from stress components, assuming steady state.

[1]Wang and Warner (1999)

# Ice Physics

**(2) anisotropic flow law**

Anisotropy important for modeling flow at nearly every location where it has been studied in detail (SDM, Byrd, Law Dome and Dome Fuji flowlines)

above)

# Ice Physics

**(3) special rheology**

**basal ice:** may be relatively softer or stiffer, depending on impurity content, impurity size, crystal size and orientation, water content, etc.

- Parameterize? Submodel? How well constrained?

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- (2,2,0) Full stokes
- (1,2,0) Higher-order
- Unsteady full stokes?
- (1,1,2) ice shelf buttressing

- (2,2,2) Rheolgy ( n >> 3, n << 3)
- (3,2,3) Damage evolution
- (3,2,3) Anisotropy
- (2,2,3) Polythermal ice

- (3,1,2) isostacy

- (1,1,2/3) Spatially variable geothermal flux; lithosphere thickness
- (1,1,2) Melting rate under ice shelves (BC)
- (1,2,2) Calving
- (1,2,3) Till evolution
- (1,1,3) Generic basal bc evolution
- (3,2,3) Sediment transport / erosion
- (2,1,1) Improved surface (energy) mass balance schemes; Accumulation patterns / redistribution

- (2,1,3) Crevasse formation (rift; ice shelf related)
- (1,2,3) Basal hydrology (related to dynamics)
- (1,1,3) Surface and englacial hydrology (params?)
- (2,1,3) Shear margin migration
- (1,3,2) Grounding lines

- (2,3,0) Adjoint model development
- (1,2,2) Model initialization

# Other

**Another incomplete list …**

-

everywhere?

# Assessment: Dynamics (cons. laws)

# Assessment: boundary conditions

# Assessment: ice physics

# Assessment: physical processes and other

# Summary

conservation of momentum

conservation of energy

conservation of mass

boundary conditions

ice physics

physical processes

other

**good** (have a handle on these)

**ok** (but room for improvement)

**bad** (need major help here)

# Equations of Stress Equilibrium (Cartesian Coordinates)

Assume static balance of forces by ignoring acceleration

$$x: \quad \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$y: \quad \frac{\partial \tau_{yy}}{\partial y} - \frac{\partial P}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = 0$$

$$z: \quad \frac{\partial \tau_{zz}}{\partial z} - \frac{\partial P}{\partial z} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial x} = \rho g$$

# Equations of Stress Equilibrium (scaled)

$$\lambda = \frac{\text{vert. length scale}}{\text{horiz. length scale}} = \frac{H}{L}$$

# Reduced-Order Approximations (scaled)

1st-order SIA: **Red** omissions ($l^2$)

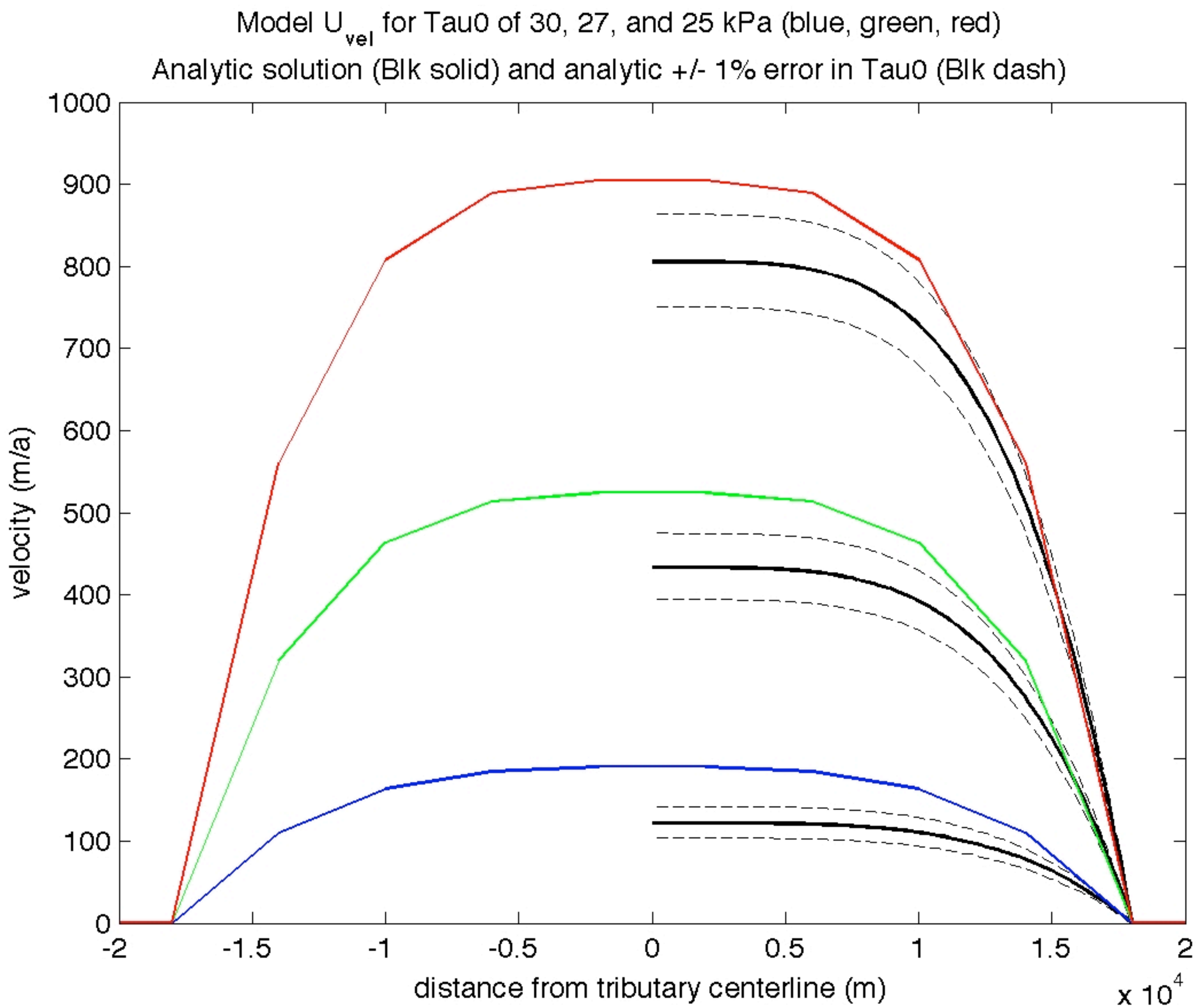
0-order SIA: **Red** + **Blue** omissions ($l$, $l^2$)

$$\hat{x}: \quad \cancel{\lambda \frac{\partial \hat{\tau}_{xx}}{\partial \hat{x}}} - \frac{\partial \hat{P}}{\partial \hat{x}} + \cancel{\lambda \frac{\partial \hat{\tau}_{xy}}{\partial \hat{y}}} + \frac{\partial \hat{\tau}_{xz}}{\partial \hat{z}} = 0$$

$$\hat{z}: \quad \cancel{\lambda \frac{\partial \hat{\tau}_{zz}}{\partial \hat{z}}} - \frac{\partial \hat{P}}{\partial \hat{z}} + \cancel{\lambda^2 \frac{\partial \hat{\tau}_{zy}}{\partial \hat{y}}} + \cancel{\lambda^2 \frac{\partial \hat{\tau}_{zx}}{\partial \hat{x}}} = 1$$

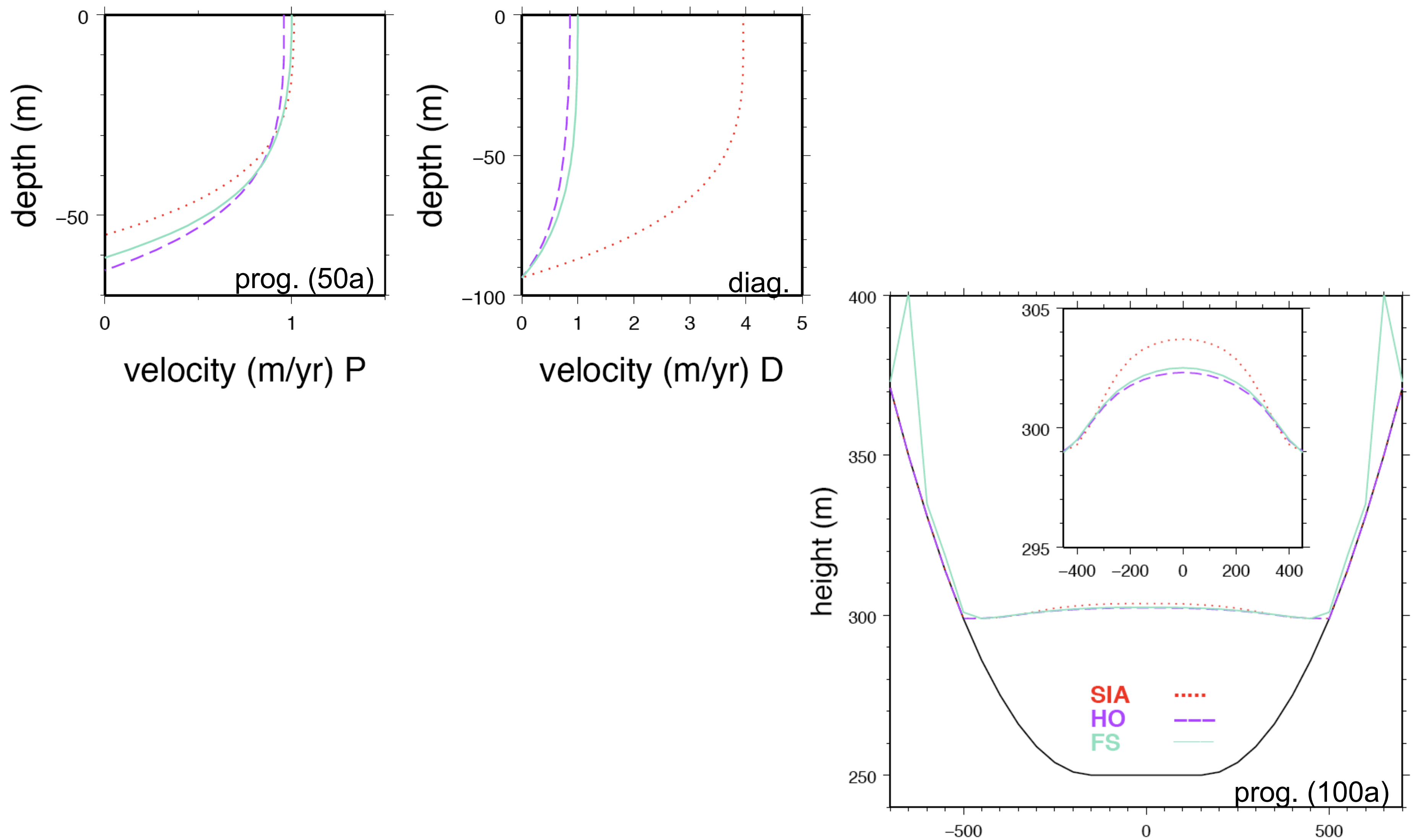
# "Sliding" over a plastic bed[1,2]

[1]Schoof (*J.Fluid.Mech.*, **556**, 2006); [2]Bueler et al. (*EGU*, 2007);



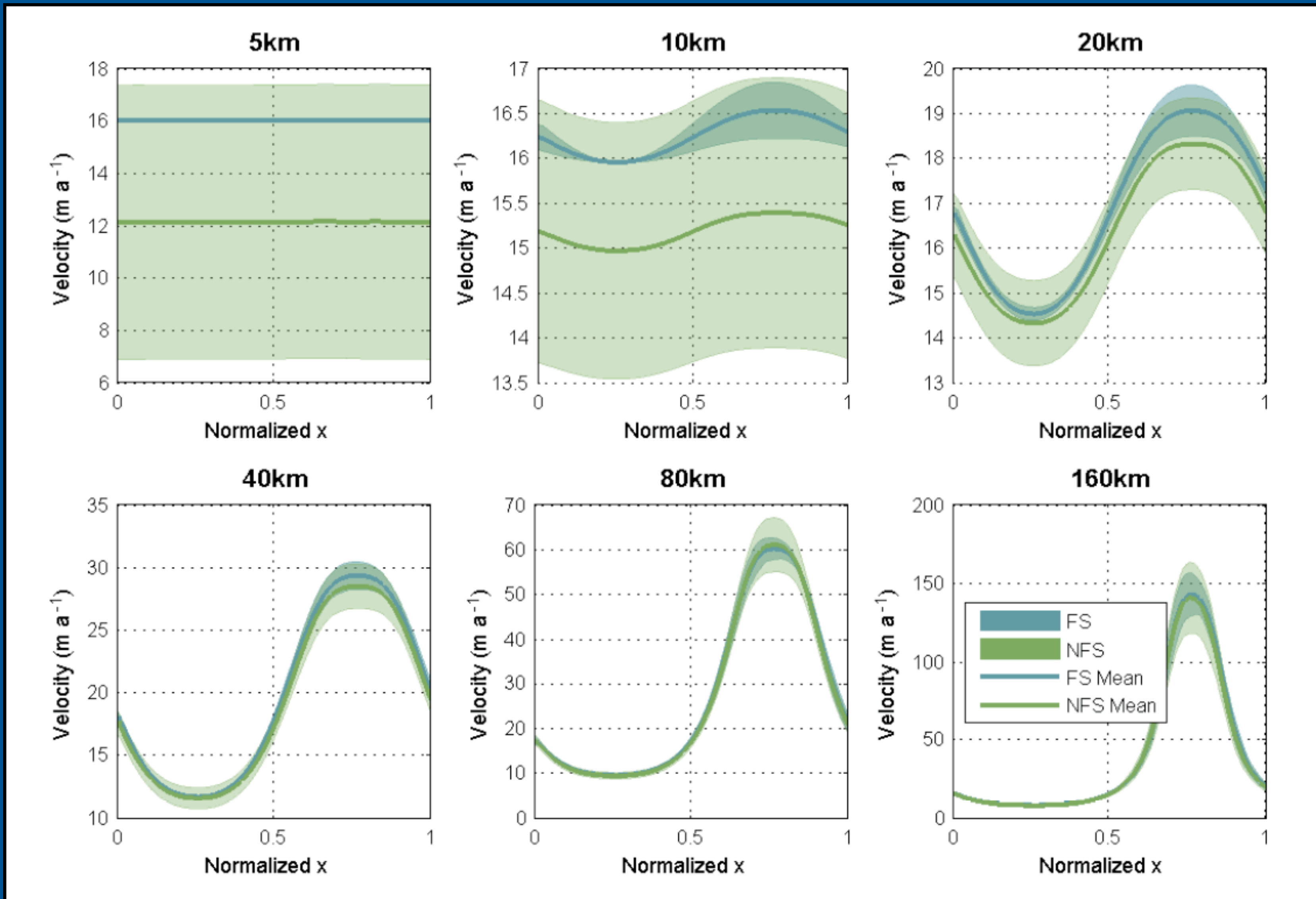
Model $U_{vel}$ for Tau0 of 30, 27, and 25 kPa (blue, green, red)
Analytic solution (Blk solid) and analytic +/- 1% error in Tau0 (Blk dash)
1000
900
800
700
600
500
400
300
200
100
0
velocity (m/a)
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
distance from tributary centerline (m)
x $10^4$

# 0-, 1$^{st}$-order SIA vs. FS




After Schafer et al. (*TCD*)

# 1st -order SIA vs. FS (sliding)




Pattyn et al. (*TC*, 2008)

# Solution Methods: cost evalution

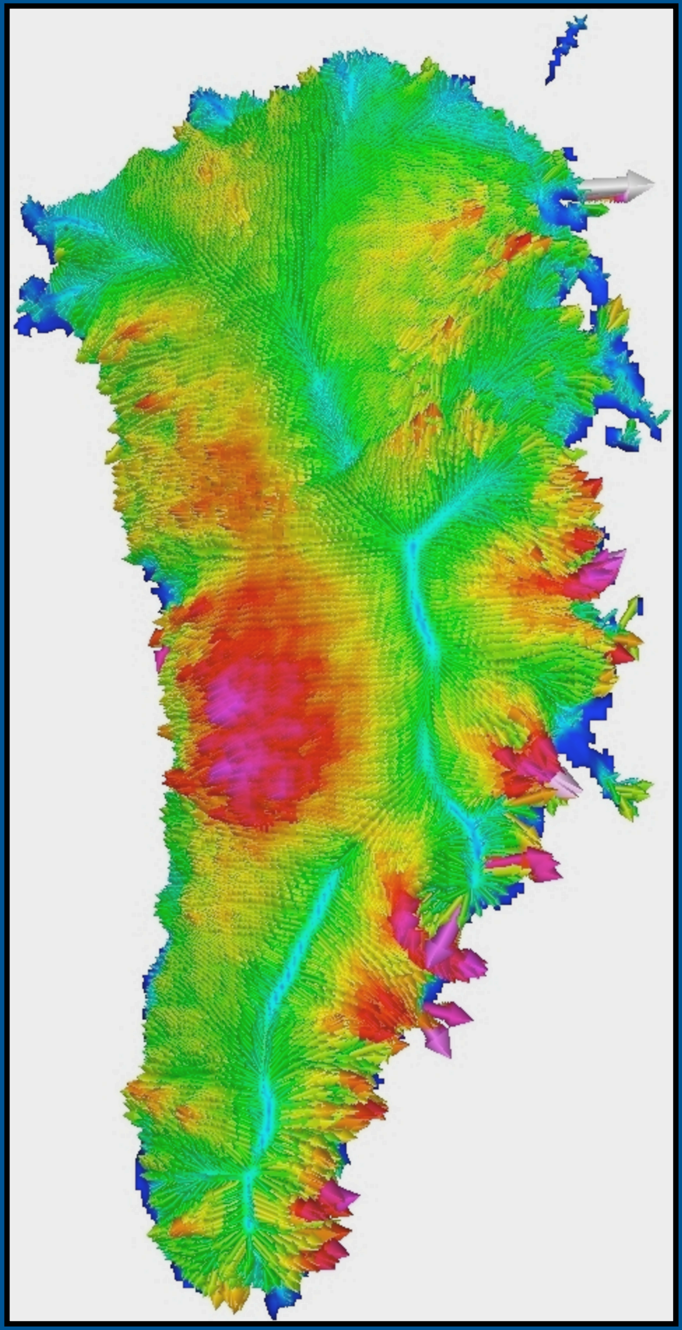
| | accuracy | vert. cont. | horiz. cont. | expense | | **score** |
|---|---|---|---|---|---|---|
| (1) | 3 | 3 | 3 | 1 | | 10 |
| (2) | 2-3 | 3 | 3 | 2 | | 11 |
| (3) | 1 | 3 | 1 | 3 | | 8 |
| (4) | 2 | 1 | 3 | 1-2 | | 8 |
| (5) | 1-2 | 2 | 2 | 2 | | 8 |
| (6) | ? | 2 | 2 | 2 | | ? |

**ELMER**[1,2] - full Stokes, FEM

- linear rheology, isothermal
- diagnostic solution only
- 10 km grid, 10 layers in vertical

For ~10 processors …

  - minutes per solve for n=1
  - hours per solve for n=3



[1]Calculation by Ralph Greve;

[2]Figure and calculation details courtesy of Thomas Zwinger *(Scientific Computing Ltd.*, Finland)

# Conservation of mass

$$(1) \quad \frac{\partial H}{\partial t} = -\nabla \cdot \left( \bar{\mathbf{u}} H \right) + \dot{b} - \dot{m} \qquad (2) \quad \frac{\partial H}{\partial t} = \nabla \cdot \left( D \nabla s \right) + \dot{b} - \dot{m}$$

For HO models:

(i) Is there a non-*ad hoc* way to define ***D*** in (2)?

(ii) Is there any reason *not* to treat (1) as a transport equation?

(iii) Is “diffusive” behavior in (2) already captured entirely by ***u*** in (1)?