A new penalty based approach to large scale modeling of Antarctica and Greenland using 2d-3d lower and higher order finite elements.

Eric Larour, Jet Propulsion Laboratory.
Eric Rignot, Jet Propulsion Laboratory.

Jet Propulsion Laboratory
California Institute of Technology

Outline.

0 - Introduction

I - Cielo parallel finite element framework.

II – Ice flow equations and penalty methods.

III – Large scale model of Antarctica and Greenland.

IV - Conclusions and perspectives.

0 - Introduction.

- Large scale modeling of ice flow over the entire Antarctic continent is a difficult problem that needs to be addressed if we want to accurately assess the impact of the continent's mass balance in global climate models.
- The challenge is twofold:
  - Sheer size of models. Antarctica in 2d, triangular mesh = 20 Million elements. Implications for parallel technologies that need to be used.
  - Range of physics that need to be addressed within the same model: grounding line migration, ice shelf calving, basal melting, surface mass balance, coupling with GCMs, etc …
- New finite element framework CIELO tries to address both challenges by providing modular design, and embedded parallel technologies. We show an example of large scale model using 2d-3d lower and higher order elements, connected using penalty methods.

I- Cielo: a new parallel FEM software.

• **JPL R&TD effort. Started in 1998. 10 man years. Goal: simulating large aperture optical systems (Terrestrial planet finders).**

• **General-purpose finite element-based computational toolset that provides fundamentally integrated thermal, structural and optical aberration capabilities**

• **Hosted in Matlab (ease of use), written in C (efficiency), parallelised using Petsc libraries and MPI communications. Deployed on Cosmos 1024 Xeon cluster at JPL.**

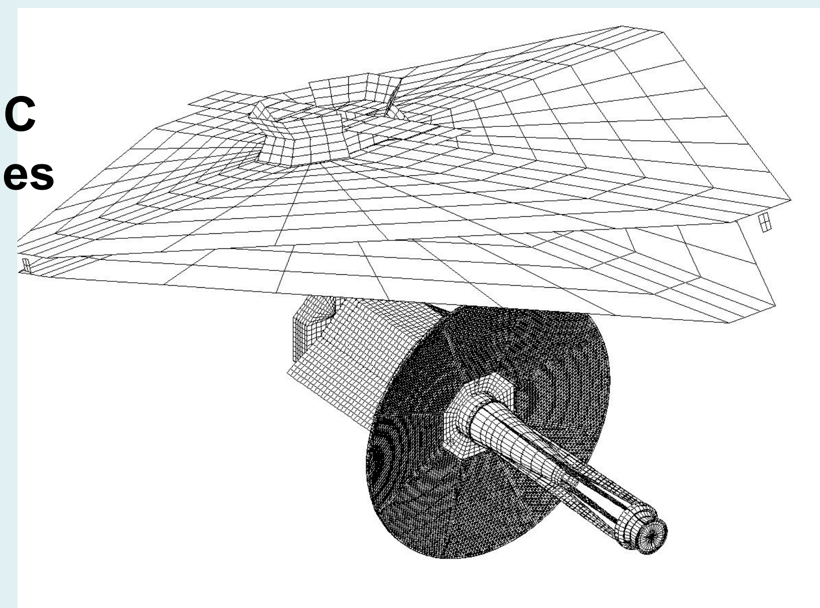
Software developpers:

Greg Moore (optimisation, architecture)

Mike Chainyk (thermal capabilities)

Claus Hoff, John Schiermeier (Element techonologies)

Eric Larour (Element and Parallel technologies)



II- Ice flow equations and penalty methods.

- **Full Navier Stokes 3d:**

$$\nabla \bullet \vec{V} = 0$$

$$\rho \frac{d\vec{V}}{dt} = \nabla \bullet \sigma + \rho \vec{g}$$

$$\rho \frac{d(c_p \theta)}{dt} = \nabla (k \nabla \theta) + \Phi$$

+

$$\sigma_{ij}{}' = 2\eta \dot{\varepsilon}_{ij}$$

$$\eta = \frac{1}{2} A(\theta)^{\frac{-1}{n}} \left(\dot{\varepsilon} + \dot{\varepsilon}_0 \right)^{\frac{(1-n)}{n}}$$

- **Neglect variational stress (T term) in the vertical direction -> problem at ice divides.**

$$\frac{\partial \sigma_{zz}}{\partial z} \approx \rho g \Leftrightarrow \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = 0$$

- **Neglect horizontal gradients of vertical velocity with respect to vertical gradients of horizontal velocity. Higher order 3d model, not always valid in upper areas of ice streams.**

$$\frac{\partial w}{\partial x} << \frac{\partial u}{\partial z}$$ **and** $$\frac{\partial w}{\partial y} << \frac{\partial v}{\partial z}$$

➢ **Neglect vertical gradients of horizontal velocity: (Shallow ice approximation -> 2d). Valid for fast flowing ice streams and ice shelves.**

$$\frac{\partial u}{\partial z} = 0 \qquad \textbf{and} \qquad \frac{\partial v}{\partial z} = 0$$

➢ **Boundary conditions:**

- **Dirichlet:**
  - **- moutains at 0 m/a velocity.**
  - **- surface temperatures from GCM.**
- **Neumann:**
  - **- dynamic pressure load at ice front.**
  - - **bedrock friction: depends on water pressure (hydrological model)**

$$\sigma_b = \eta . N_{eff}^{\ p} \sqrt{u^2 + v^2}^{\,q}$$

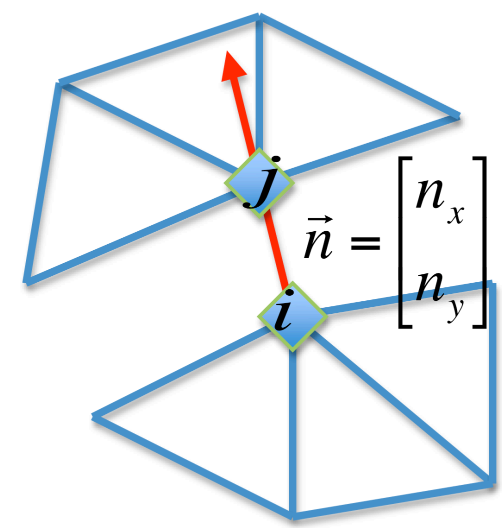
  - - **geothermal flux.**

How to handle nesting of these elements: using computational contact mechanics -> penalty contact.

$$Kij = \lambda \begin{bmatrix} n_x^2 & n_x n_y & -n_x^2 & -n_x n_y \\ n_x n_y & n_y^2 & -n_x n_y & -n_y^2 \\ -n_x^2 & -n_x n_y & n_x^2 & n_x n_y \\ -n_x n_y & -n_y^2 & n_x n_y & n_y^2 \end{bmatrix}$$

Peter Wriggers, Computational Contact Mechanics.

where λ is a penalty parameter, 10^2*norm(K,2) and Kij is a stiffness matrix between grids i and j, for degrees of freedom x and y.



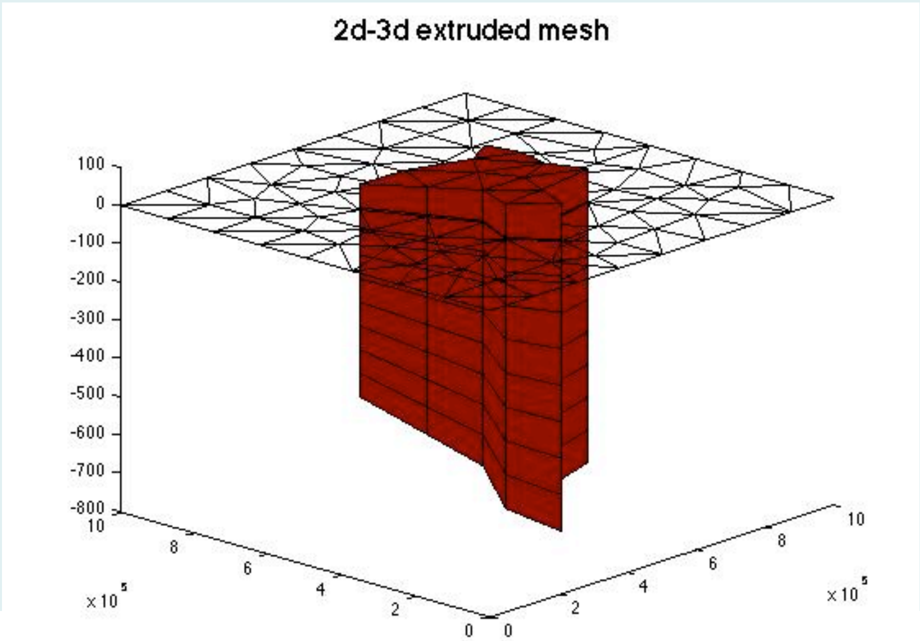
- For 2d-3d nesting: two normals (1,0,0) and (0,1,0).
- For each normal, the penalty stiffness for x and y degrees of freedom, between grids i and j:

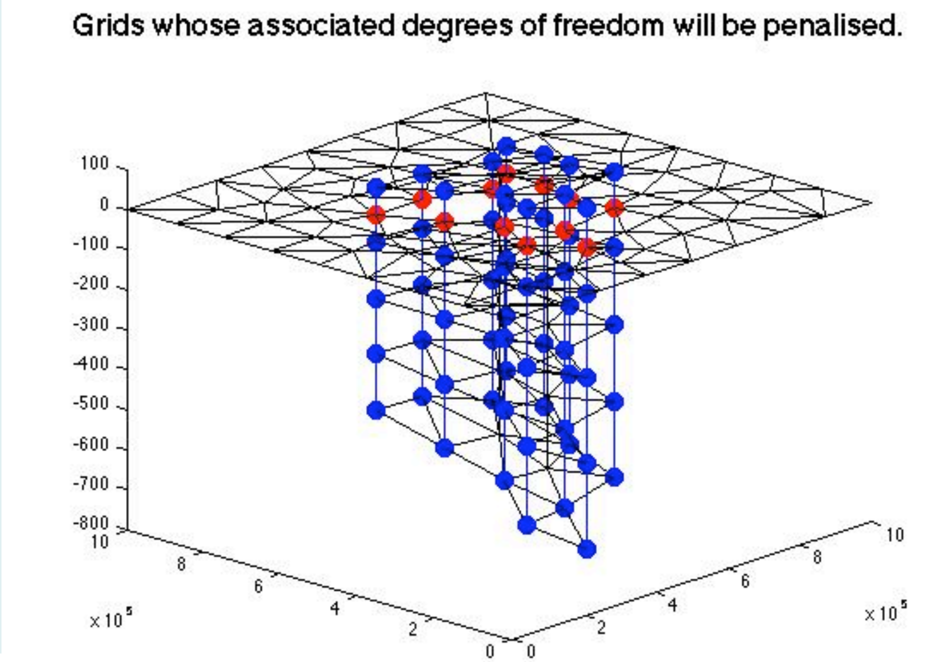
$$Kij = \begin{bmatrix} \lambda & -\lambda \\ -\lambda & \lambda \end{bmatrix}$$

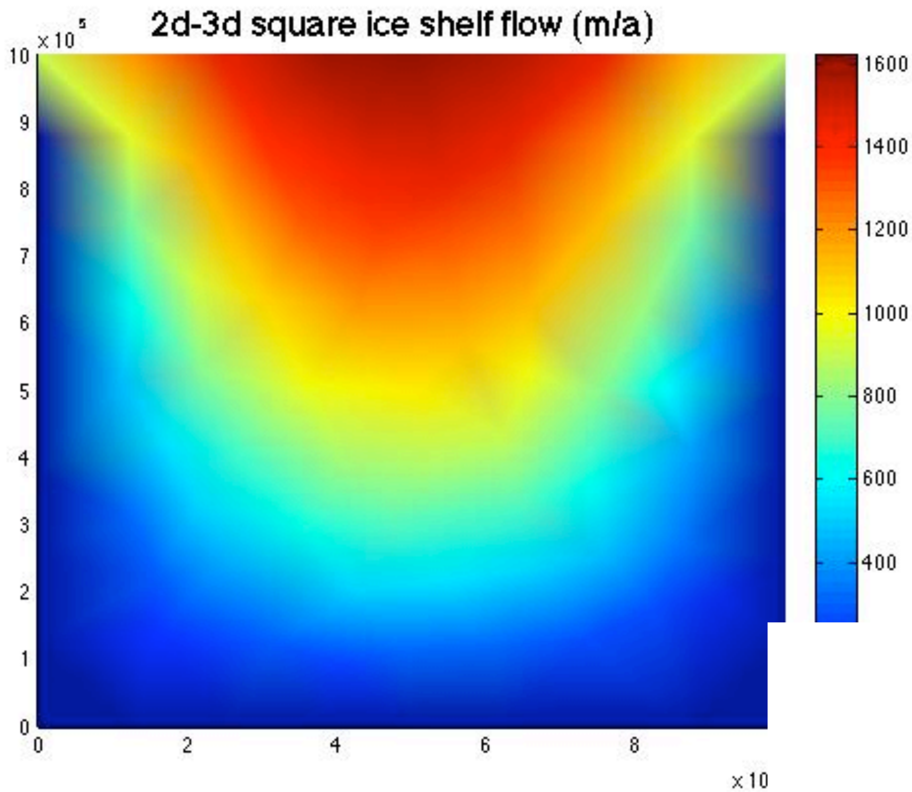
- This is equivalent to 2 MPC (Multi point constraint) relationships that lower the rank of the global stiffness by 2.

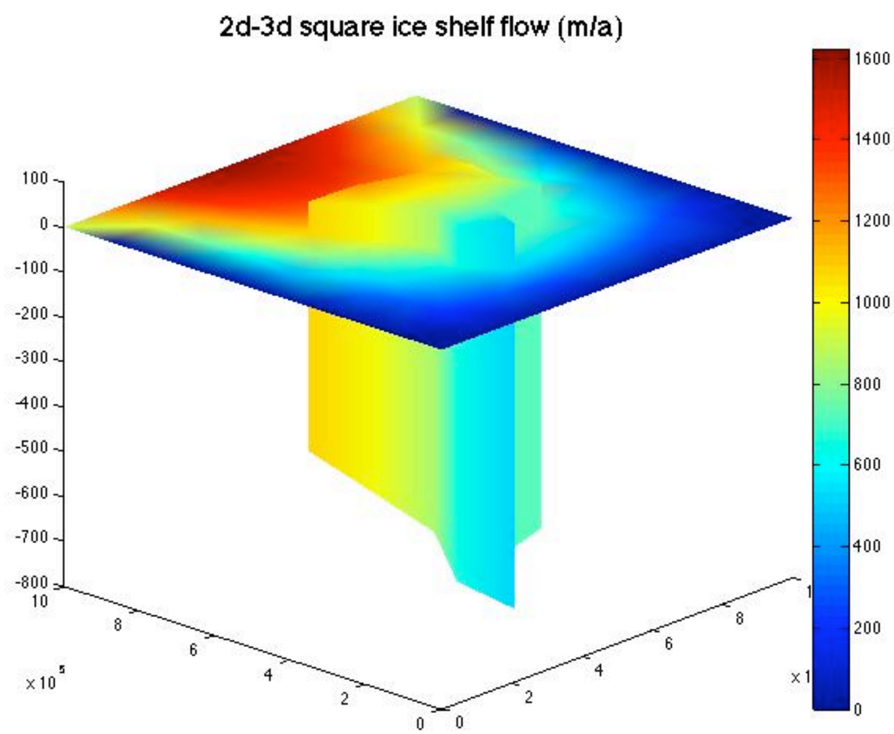
$$Vx_i = Vx_j$$

$$Vy_i = Vy_j$$






2d-3d square ice shelf flow (m/a)
x 10^5
10
9
8
7
6
5
4
3
2
1
0
0
2
4
6
8
x 10
1600
1400
1200
1000
800
600
400


2d-3d square ice shelf flow (m/a)
100
0
-100
-200
-300
-400
-500
-600
-700
-800
10
8
6
4
2
0
x 10^5
0
2
4
6
8
1600
1400
1200
1000
800
600
400
200
0

III- Large scale model of Antarctica and Greenland.

Input parameters for large scale models of Antarctica:

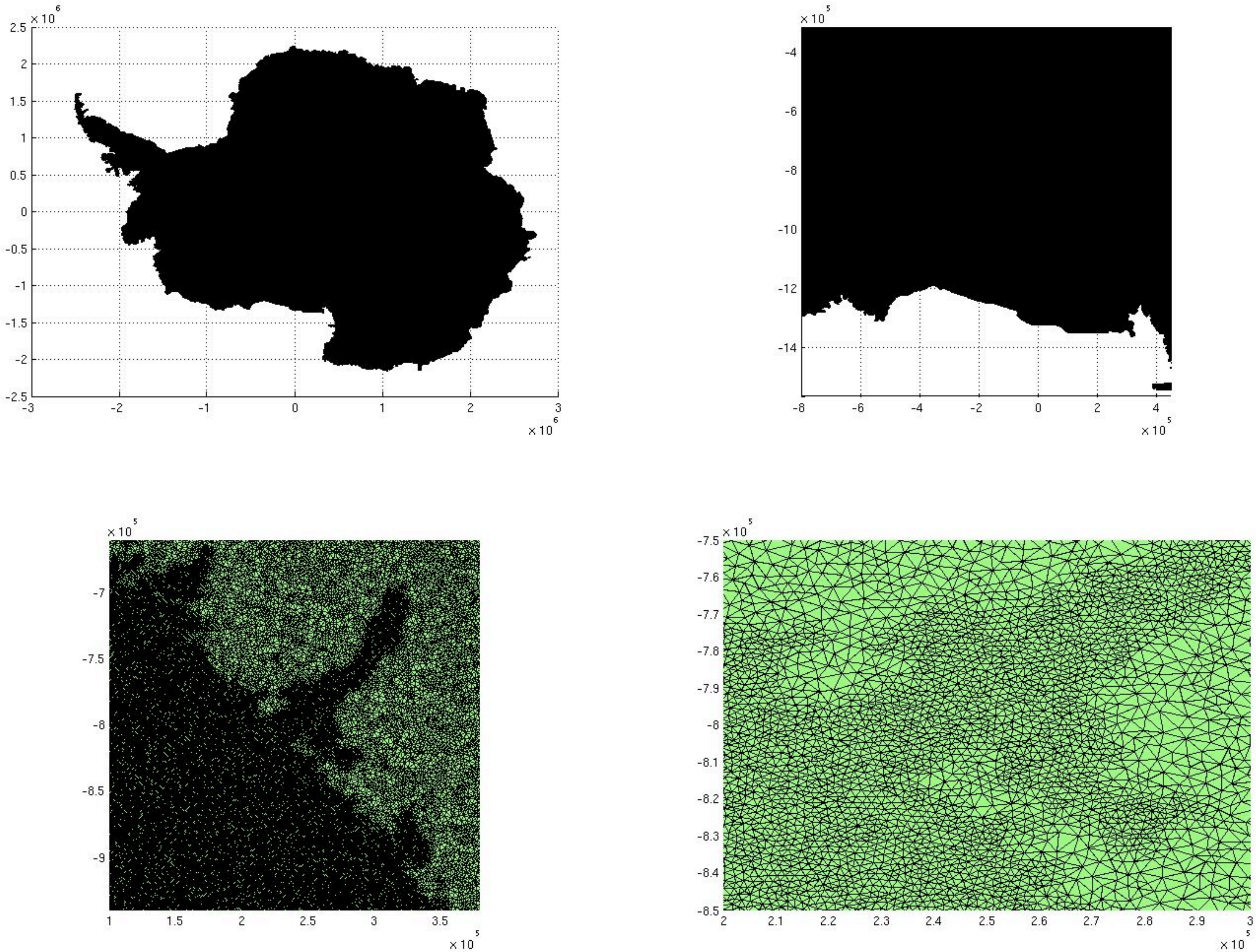
- **Firn Layer: van den Broeke, M.R.,** Towards quantifying the contribution of the Antarctic ice sheet to global sea level change*. Journal of Physics. IV France,* **2006** *(139) 170-187*

- ***Temperatures: Giovinetto, M.B., N.M. Waters, and C.R. Bentley,*** Dependence of Antarctic surface mass balance on temperature, elevation and distance to open ocean*, Journal of Geophysycal Research,* ***1990,*** *95, 3517-3531*

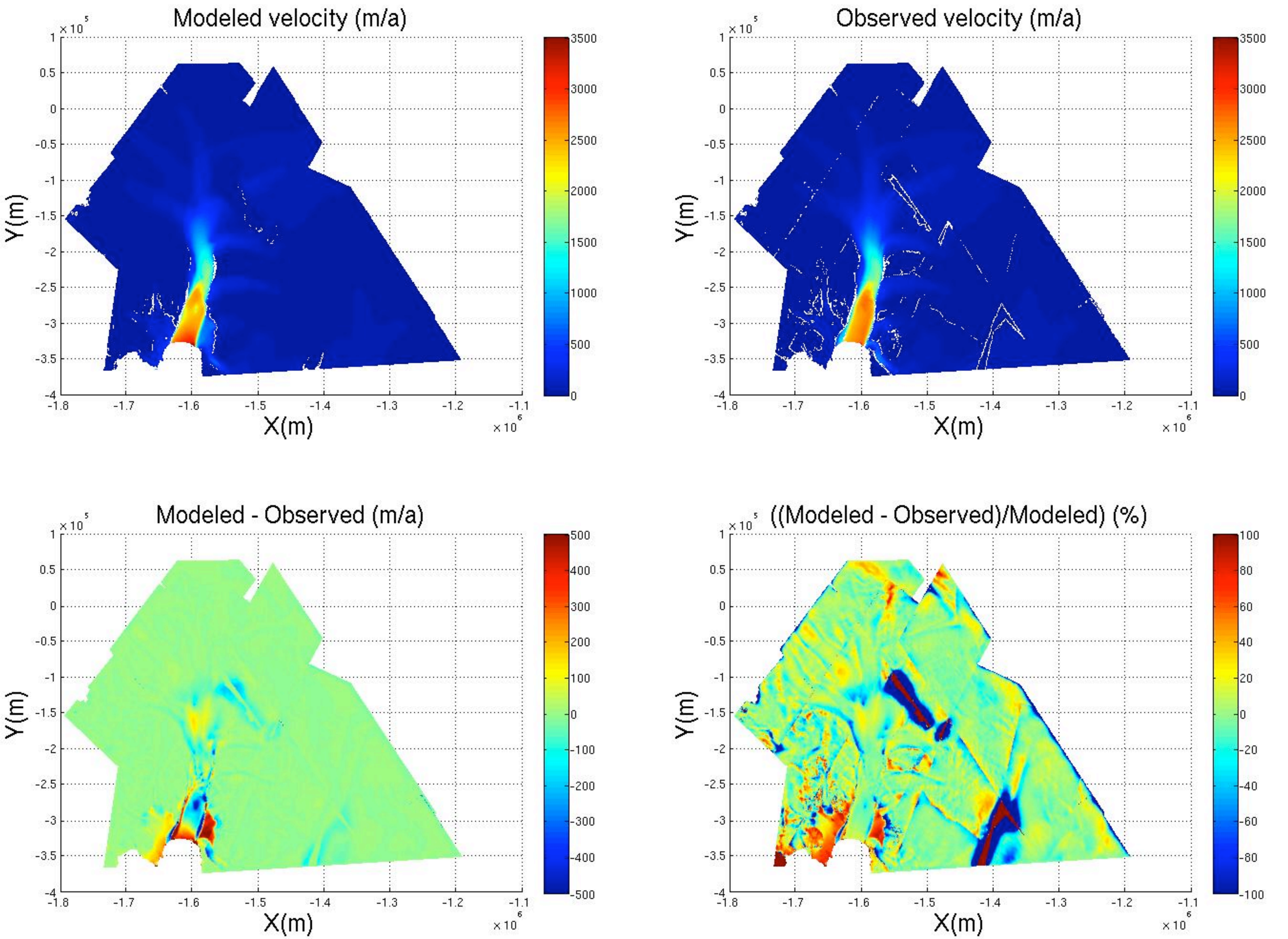
- **Surface: Bamber, J. L., unpublished**

- **Thickness: Lythe,M.B., D.G. Vaughan and Consortium BEDMAP**, BEDMAP: A new ice thickness and subglacial topographic model of Antarctica, *Journal of Geophysical Research,* ***2001****, 106 (B6), 11,335-11,352*

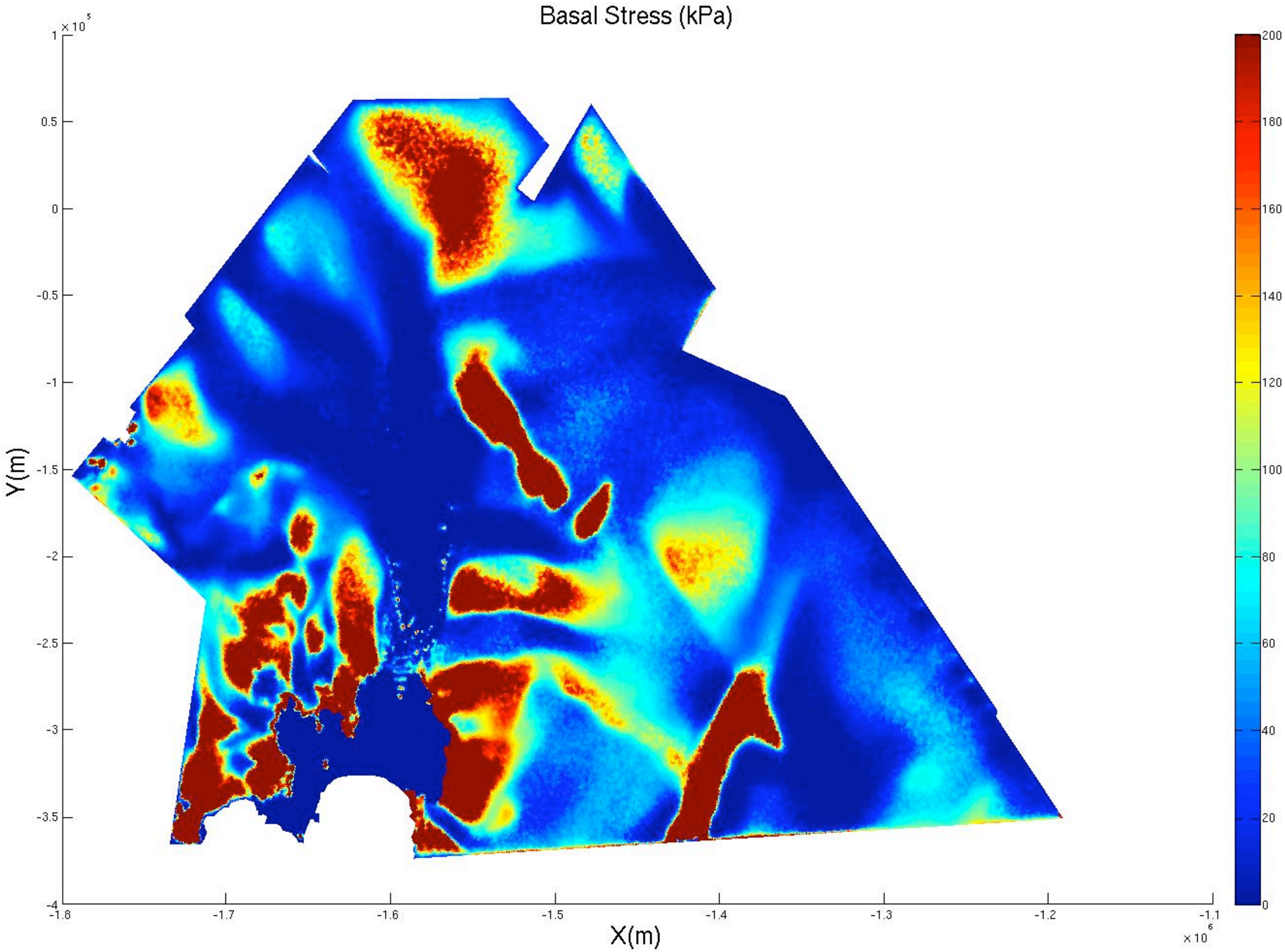
- **Grounding Line, Ice Front, Ice Rises: Rignot unpublished.**



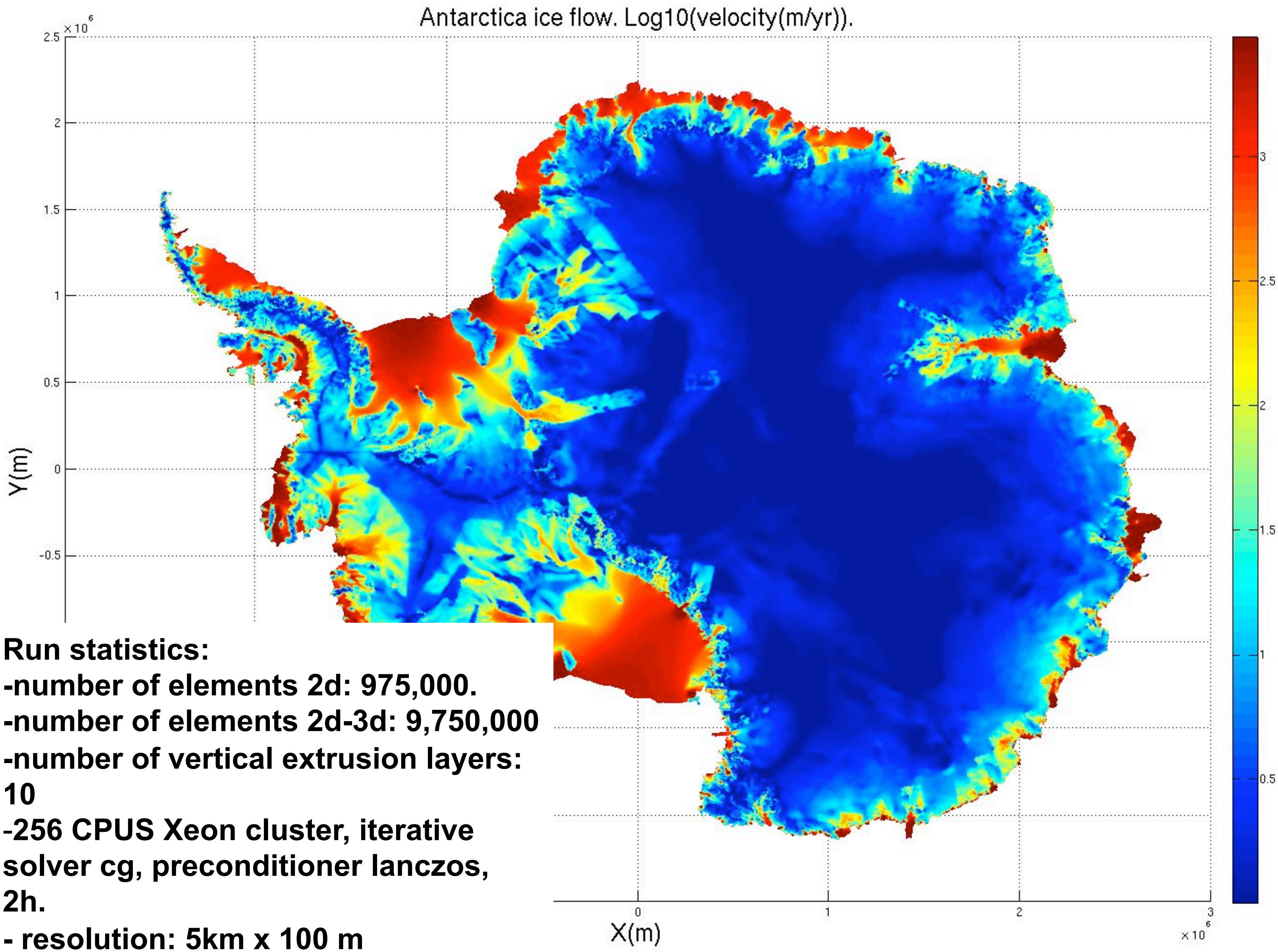
×10^6
2.5 2 1.5 1 0.5 0 -0.5 -1 -1.5 -2 -2.5
-3 -2 -1 0 1 2 3
×10^6
×10^5
-4 -6 -8 -10 -12 -14
-8 -6 -4 -2 0 2 4
×10^5
×10^5
-7 -7.5 -8 -8.5 -9
1 1.5 2 2.5 3 3.5
×10^5
-7.5 ×10^5
-7.6 -7.7 -7.8 -7.9 -8 -8.1 -8.2 -8.3 -8.4 -8.5
2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
×10^5



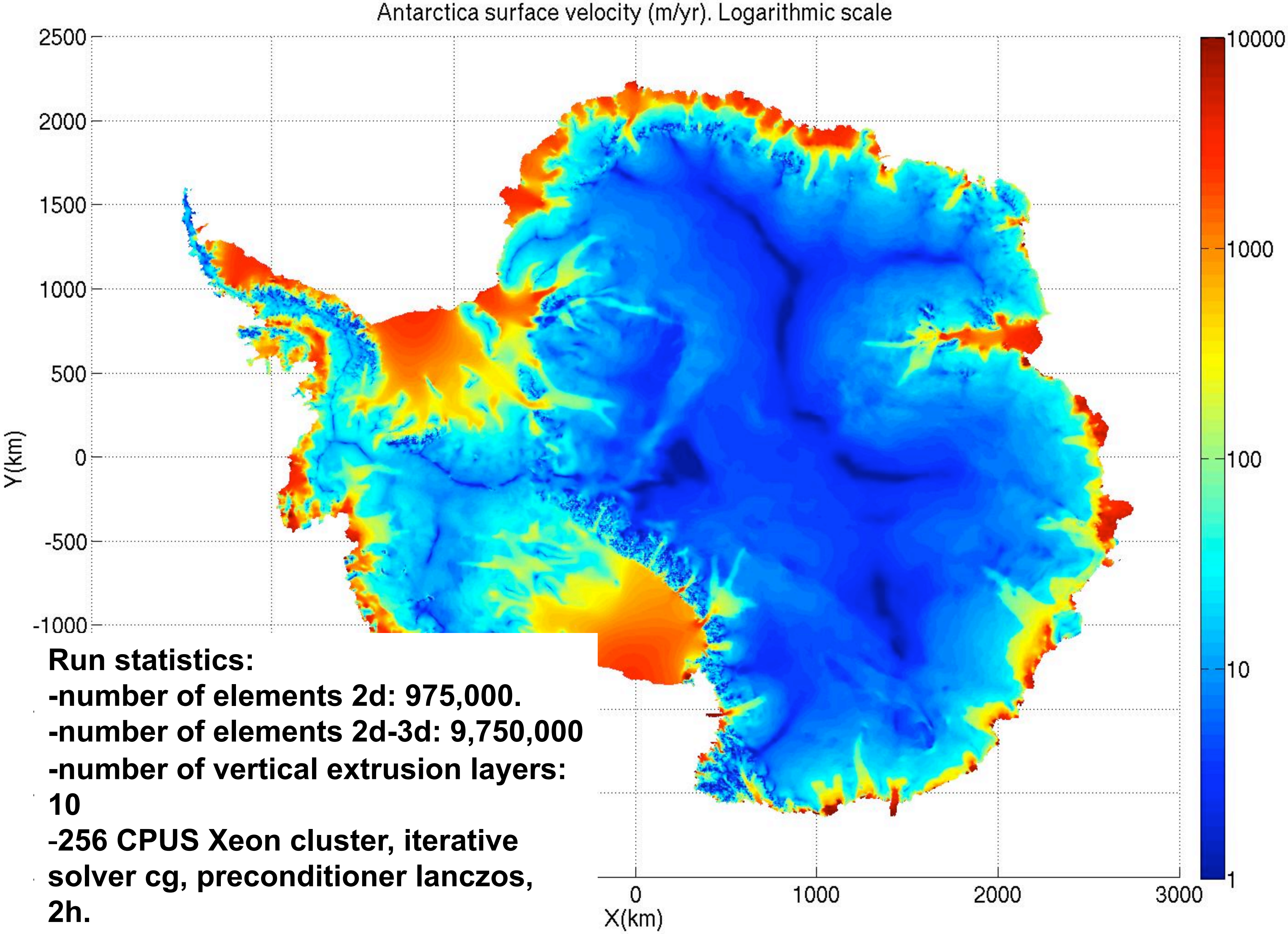
Modeled velocity (m/a)
Observed velocity (m/a)
Modeled - Observed (m/a)
((Modeled - Observed)/Modeled) (%)
X(m)
Y(m)
× 10^5
× 10^6



Basal Stress (kPa)
Y(m)
X(m)
× 10^5
× 10^6
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-1.8
-1.7
-1.6
-1.5
-1.4
-1.3
-1.2
-1.1
200
180
160
140
120
100
80
60
40
20
0

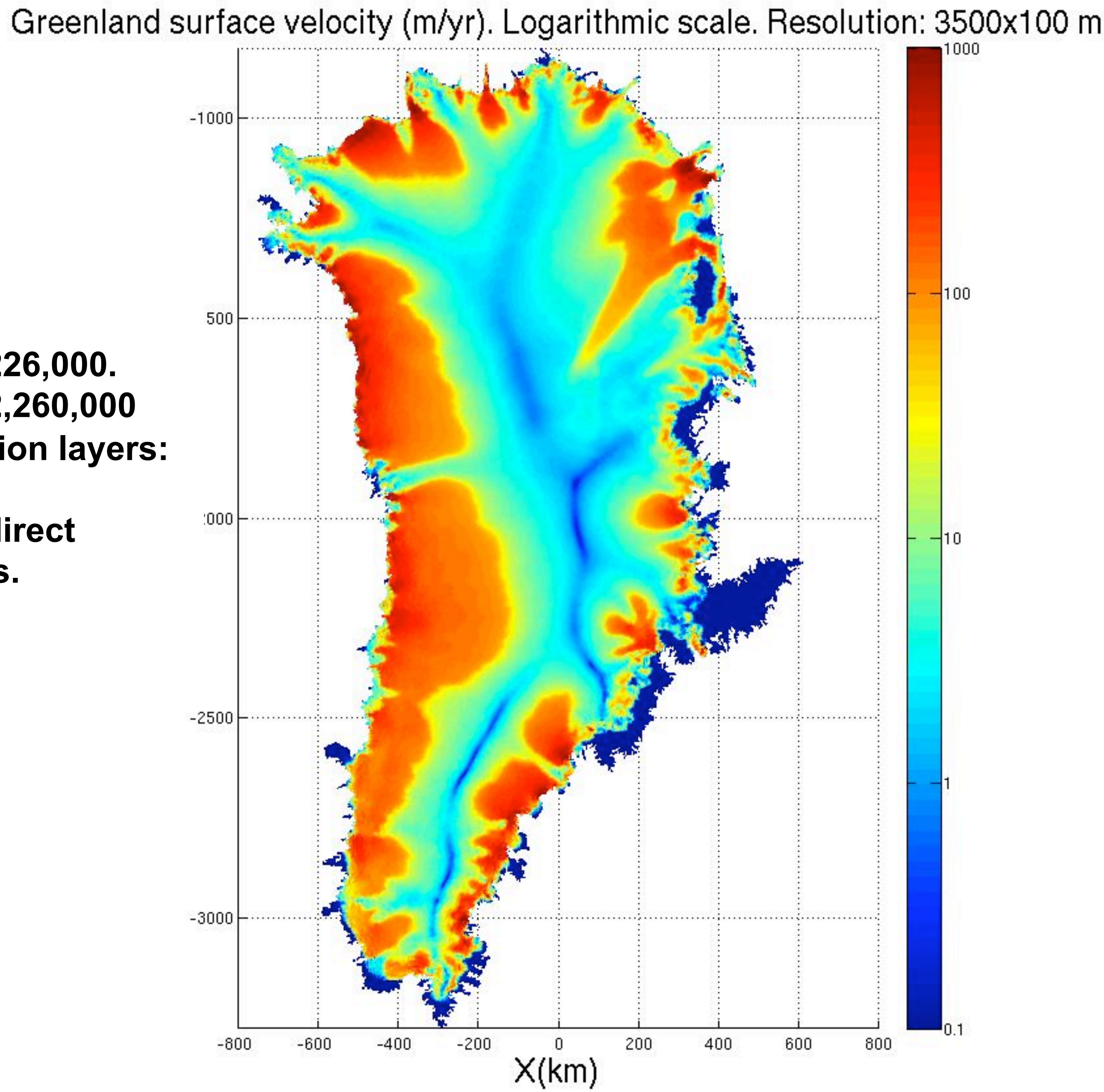


Run statistics:
-number of elements 2d: 975,000.
-number of elements 2d-3d: 9,750,000
-number of vertical extrusion layers: 10
-256 CPUS Xeon cluster, iterative solver cg, preconditioner lanczos, 2h.
- resolution: 5km x 100 m



Run statistics:
-number of elements 2d: 975,000.
-number of elements 2d-3d: 9,750,000
-number of vertical extrusion layers: 10
-256 CPUS Xeon cluster, iterative solver cg, preconditioner lanczos, 2h.
-resolution: 5km x 100 m

Run statistics:
-number of elements 2d: 226,000.
-number of elements 3d: 2,260,000
-number of vertical extrusion layers: 10
-256 CPUS Xeon cluster, direct MUMPS solver, 45 minutes.
- resolution: 3500x100 m



Conclusions and perspectives.

Conclusions:

- Large scale diagnostic models for Greeland and Antarctica reaching acceptable resolutions.
- Penalty methods allow for multi-scale modeling.

Perspectives

- Implement prognostic modelling.
- Couple with GCM.
- Full-stokes at ice divide and grounding lines.
- Out-of-core direct solvers, physical preconditioners.

Backup slides.

Evolution models for ice shelves depend heavily on rheology of ice (ice flow) and rifting processes (calving rates).

Rheology can be investigated using inverse methods (no direct way of measuring rigidity B).

Evolution models then produce better ice flow, but rifting processes still have to be accounted for, using Linear Elastic Fracture Mechanics.

Mac Ayeal's forward ice flow model and inverse control methods:

$$\frac{\partial}{\partial x}\left(2\nu H\left(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right)+\frac{\partial}{\partial y}\left(\nu H\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)-\rho g H\frac{\partial z_s}{\partial x} = 0$$

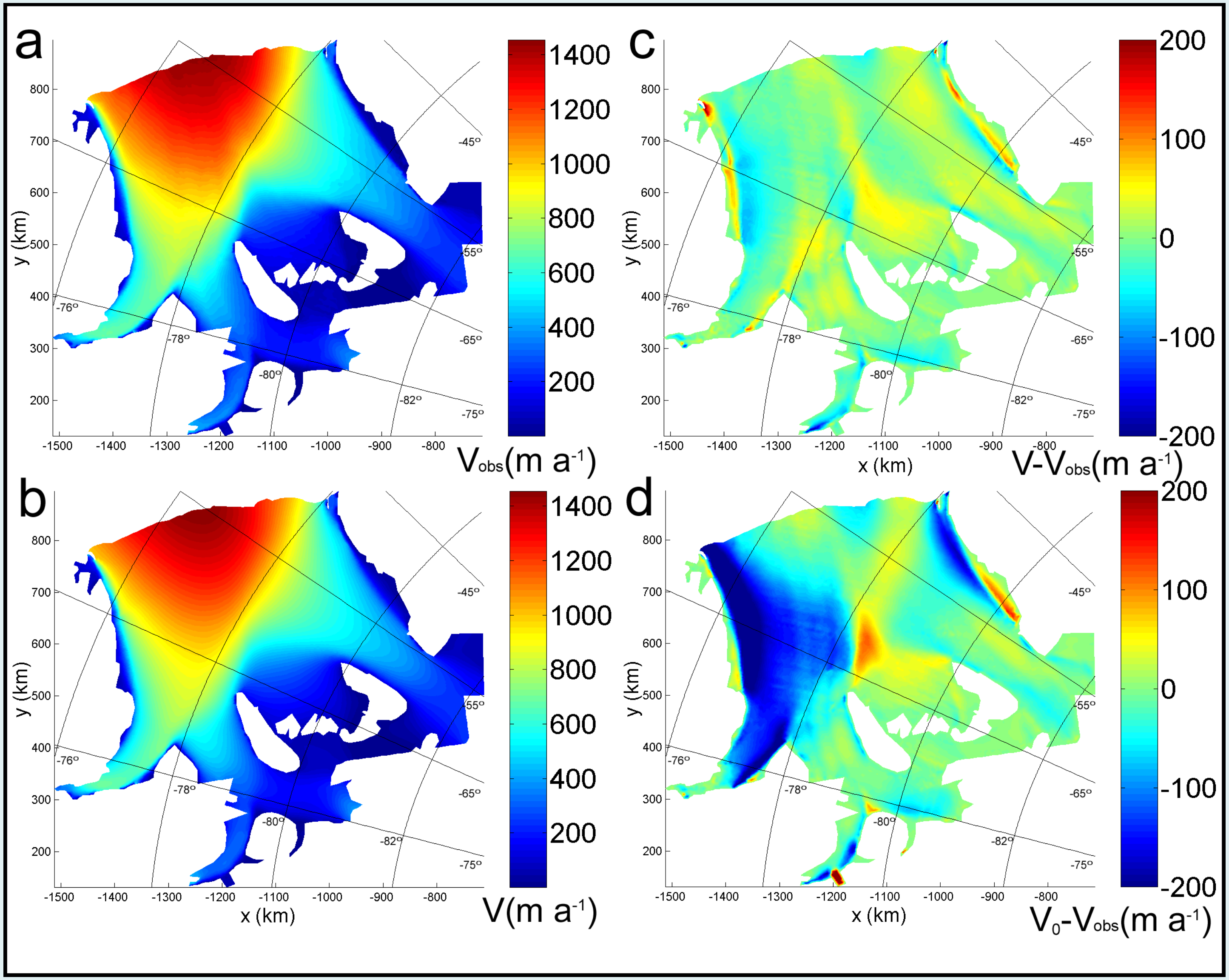
$$\frac{\partial}{\partial y}\left(2\nu H\left(2\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)\right)+\frac{\partial}{\partial x}\left(\nu H\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)-\rho g H\frac{\partial z_s}{\partial y} = 0$$

$$\nu = \frac{B}{2\left\{\frac{\partial u}{\partial x}^2+\frac{\partial v}{\partial y}^2+\frac{1}{4}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\right\}^{\frac{n-1}{2n}}}$$

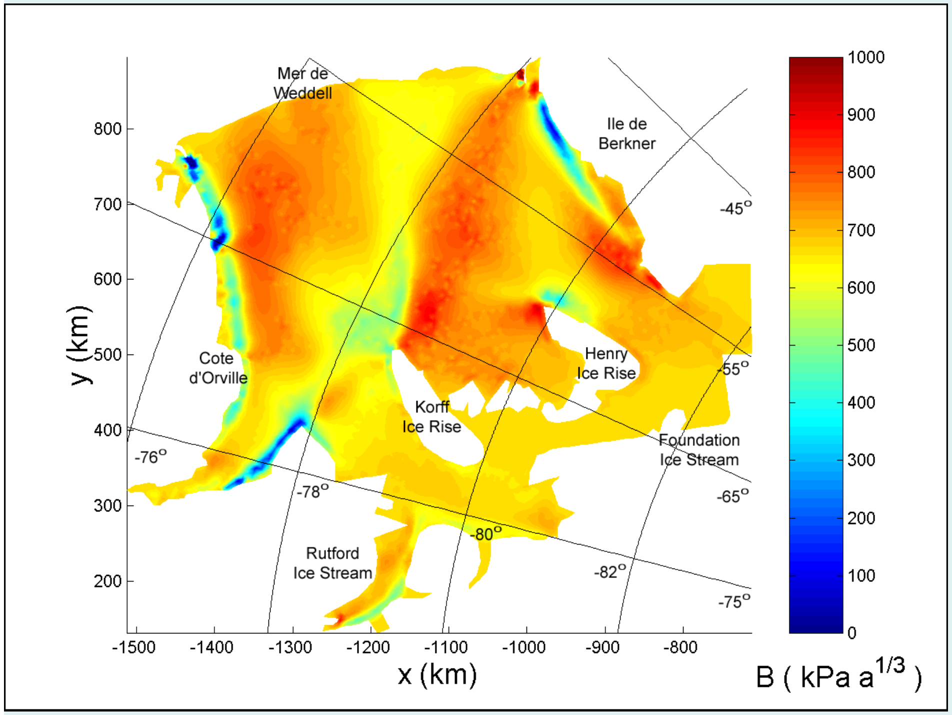
Iterate on misfit J between observed velocities and modeled velocities:

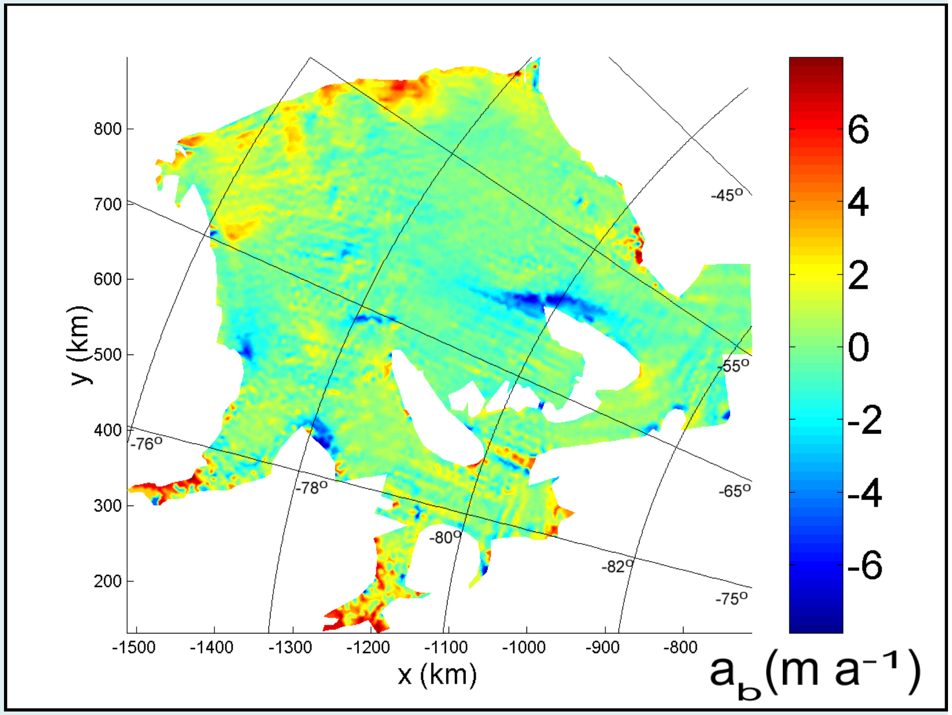
$$\begin{aligned}
J' &= \int\int_\Gamma \frac{1}{2}\{(u-u_d)^2+(v-v_d)^2\}dxdy \\
&+\int\int_\Gamma \lambda(x,y)\left\{\frac{\partial}{\partial x}\left(2\nu H\left(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right)+\frac{\partial}{\partial y}\left(\nu H\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)-\rho g H\frac{\partial z_s}{\partial x}\right\}dxdy \\
&+\int\int_\Gamma \mu(x,y)\left\{\frac{\partial}{\partial y}\left(2\nu H\left(2\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)\right)+\frac{\partial}{\partial x}\left(\nu H\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)-\rho g H\frac{\partial z_s}{\partial y}\right\}dxdy
\end{aligned}$$

Optimization results for inversion on rigidity B.



Inversion results for rigidity coefficient B.





Abstract

Large scale modeling of ice flow over the entire Antarctic continent is a difficult problem that needs to be addressed if we want to accurately assess the impact of the continent's mass balance in global climate models.

The challenge is twofold. First, we need to address the sheer size of such a model, and the implications for the parallel technologies that need to be used. Second, the range of physics that need to be described is wide (grounding line migration, basal melting, firn densification, subglacial networks, etc ...).
In this work, we present a new finite element framework called Cielo, and a new hybrid 2d-3d mechanical model that try to address both issues.
Cielo is a general purpose finite element model that can be used to model most types of physics, while seamlessly integrating parallel technologies. Using this framework, we have implemented Doug MacAyeal's lower order 2d shelf-stream and integrated it with Frank Pattyn's higher order 3d model. Lower order models can be used on most of Antarctica's surface. Higher order models can be used where the shallow ice approximation is not valid anymore. Both types of formulations can be integrated into one common model using penalty methods.
The end result is a model that can adapt its formulation to a local set of physics, while taking advantage of the reduction of computational needs that comes with using lower order 2d elements.
We present our results on Antarctica, using our model, and a mesh with a level of refinement that reaches the 1km mark wherever it is needed. We show realistic results for the pattern of velocity distribution compared with InSAR data.