# Grounding line movement and ice shelf buttressing in marine ice sheets: an adaptive approach

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# Outline

- Marine Ice Sheet and Grounding Line Dynamics
- My Solution
  - Numerics (Adaptive finite elements)
  - Validation in 1D and 2D
  - Effect of buttressing and ice rises on Marine Ice Sheet Instability (and movies, if time permits)

We are interested in modeling marine ice sheets..

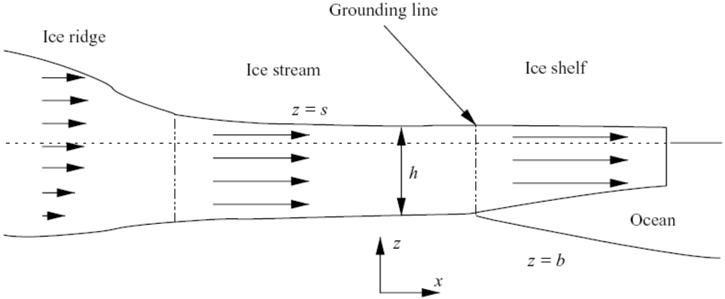


Figure: Schoof, 2006

# Hierarchy (or "Zoology") of ice models

Several models approximate Stokes flow with increasing complexity; In this study we use a 2D model

| **Type** | **Flow regime** | **Along-flow direction** | **Transverse direction** | **Vertical** | **Example** |
|---|---|---|---|---|---|
| 1D | plug *or* shear | resolved | ignored | parameterized | *Vieli and Payne* 2005 |
| quasi-2D | plug | resolved | parameterized | parameterized | *Dupont* 2004 |
| 2D-planar | *N/A* | resolved | ignored | resolved | *Pattyn et al* 2006 |
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# Shelfy-Stream Equations

- Momentum Balance (diagnostic):

$$\nabla \cdot (h\nu \vec{D}) + \vec{\tau_b} = \rho g h \nabla (b + h),$$

$$D_{ij} = 2\dot{\epsilon}_{ij} + 2(\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy})\delta_{ij},$$

$$\nu = \frac{A^{-\frac{1}{n}}}{2}\left(u_x^2 + v_y^2 + u_x v_y + \frac{1}{2}(u_y + v_x)^2\right)^{-1/3}$$

- Mass Balance (prognostic):

$$h_t + \nabla \cdot (\vec{u} h) = a$$

- Basal stress closure:

$$\vec{\tau}_b = \begin{cases} -C|u|^{m-1}\vec{u} & h > -\frac{\rho_w}{\rho} z_{bed} \\ 0 & o.w. \end{cases}, \quad m = \frac{1}{3}, \quad C = \text{constant}.$$

# Boundary Condition

At the ice shelf front, the b.c. arises from a pressure imbalance; effectively, the shelf is being "pulled" seaward:

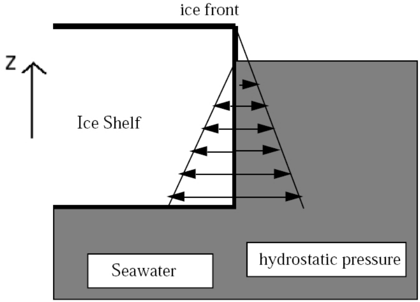



Figure: MacAyeal, "Lessons in Ice-Sheet Modeling"

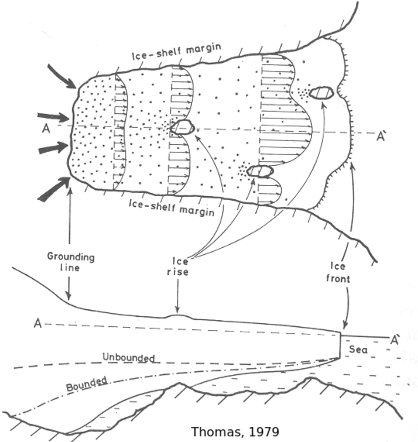
$$\int (\vec{\sigma} \cdot \vec{n} - p)\, dz = -\frac{\rho_w g}{2} (\frac{\rho}{\rho_w} h)^2 \vec{n}$$

*or*

$$h\nu \vec{D} \cdot \vec{n} = \frac{\rho g}{2} (1 - \frac{\rho}{\rho_g}) h^2 \vec{n}$$

# Buttressing

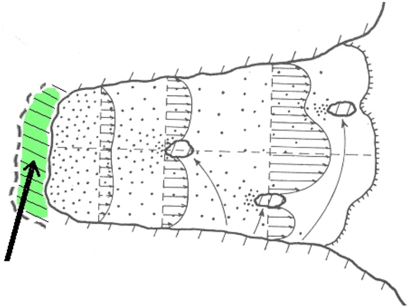
- How this "pulling" stress is is transmitted through the shelf, and how much is taken up by the margins (and possible ice rises/rumples), determine stress conditions along the grounding line.




Thomas, 1979

# Grounding zone

- The stress not taken up by margins and rises must be balanced by the "extra" basal stress in the grounded portion (above that required to balance gravity locally - loosely, $\vec{\tau}_b > \rho g h \nabla s$)
- The area where this is the case may not extend far from the G.L. in which case high sliding velocities are required in this zone



# Numerical model

The Shelfy Stream equations are solved using finite elements:

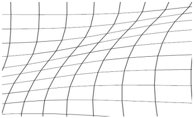
- The momentum balance is solved for $u, v$ using bilinear basis functions on rectangular cells. The nonlinear nature means the solution must be iterative ($h$ is held constant during the iterations).
- $h$ is defined as piecewise constant and the evolution of $h$ is solved by finite volume (*technically* a zero-order discontinuous Galerkin scheme).

The above two steps constitute 2 parts of a single timestep. The third is *mesh adaption*.

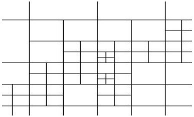
# Mesh Adaption

In order to provide the high resolution required when the Grounding zone is small compared to the domain but avoid crippling computational expense, 2 different modes of mesh adaption were implemented (and evaluated):

- **Moving Mesh** (also known as *r*-refinement) - gridpoints moved, connectivity and # of cells remain constant



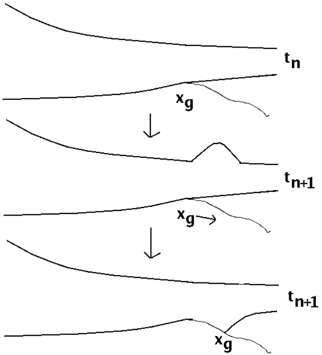
- **Adaptive Refinement** (*h*-refinement) - dividing and merging of cells - “hanging node” issues



# Representing grounding line change

And how do I actually move the grounding line?

- Movement of grounding line is completely diagnostic
- After each timestep (i.e. after $h$ is evolved and the mesh is adapted), the floatation condition ($h > -\frac{\rho_w}{\rho} z_{bed}$) is evaluated at each cell

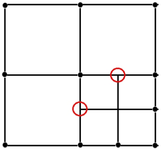


# Technology: deal.II library

*D*ifferential *E*quations *A*nalysis *L*ibrary; http://dealii.org

- Finite element software package; just a *little* higher-level than Libmesh
- Optional to replace lin alg objects by PETSc wrappers - easy to parallelize
- Lin. systems solved by conj. grad. with jacobi or block-jacobi precond.

- clever way of dealing with "hanging nodes" by linear constraints: values are slaved to neighboring nodes



# 1D Validation

- Despite recent theoretical advances (*Schoof*, 2006) there are still some loose ends, as pointed out by *Vieli and Payne* (2005), such as resolution dependence and inability to find steady states
- There is (or will be) a standard intercomparison for 1D models (MISMIP), which includes comparison with analytic results
- I will demonstrate the ability of my Moving Mesh model to reproduce such results, as well as its effectiveness with very few gridpoints while a uniform mesh fails with many gridpoints

# MISMIP (experiment 3)

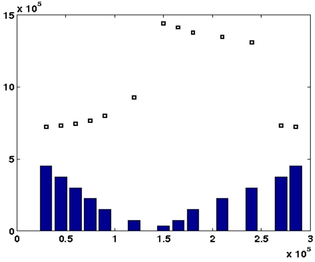



Figure: $x_g$ versus $t$

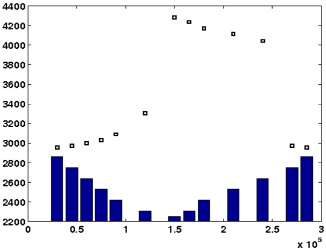



Figure: $h$ at divide versus $t$

- Experiment details: ice sheet grown from scratch on bedrock with sill, Glen's Law constant (blue bars) changed every 15-30kA
- Steady grounding line and divide thickness predicted by the quasi-analytic solution of *Schoof* (2006) shown (squares)

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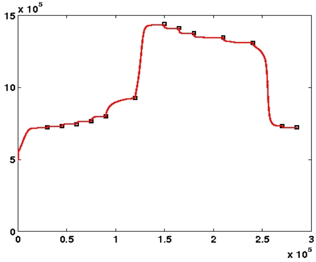


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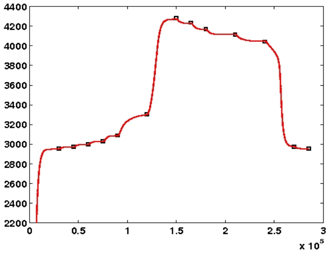



Figure: $h$ at divide versus $t$

- *deal.ii* model (moving mesh, 400 gridpoints) can be effectively 1D (no-stress on sides, bedrock only varies along-flow)

# What do we get from mesh adaption?

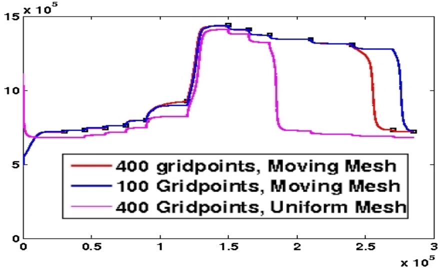



Figure: $x_g$ versus $t$

- With 400 Gridpoints, a non-adaptive mesh compares poorly with an a Moving Mesh
- However, a Moving Mesh does much better with only 100 gridpoints

# What about Adaptive Refinement in 1D?

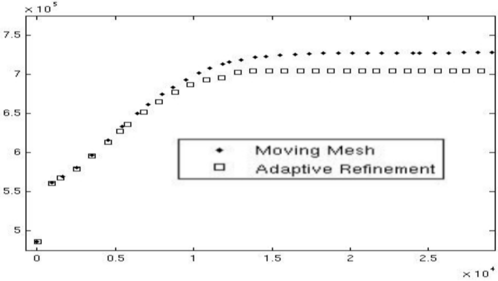



Figure: $x_g$ versus $t$

- grounding line movement from first step from MISMIP experiment
- Adaptive Refinement model seems to get "stuck" even though it should advance

# 2D Domain

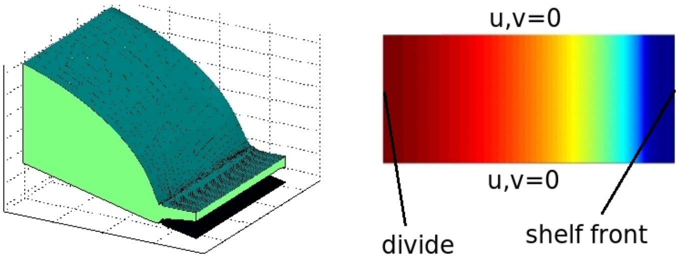



Figure: Relief and Top-Down View

2D model domain: no-slip margins and an Ice Divide at the left. The entire domain is ice-covered and the calving front is fixed. Accumulation is constant and uniform (and positive).

# 2D Experiments

- With no analytic solutions for 2D, we need to convince ourselves that the model is solving the equations satisfactorily
- Key results of Validation/"Sanity Check":
  - With a moving mesh, results are convergent w.r.t. mesh size, and low-buttressing limit is consistent with 1D solution
  - Neither a fixed mesh nor an Adaptively-Refined mesh exhibit such behaviour at comparable cost

(for more details, come to the thursday talk...)

# Buttressing/Stability Experiment

- But we seek to say something more concrete about buttressing
- It is known that, without buttressing, a foredeepened bed is unstable
- A small retreat of the G.L. results in higher G.L. flux, which leads to thinning at the G.L. and more retreat (*Weertman*, 1973)..

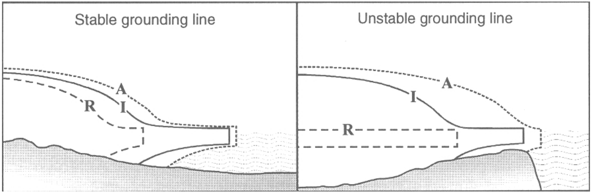
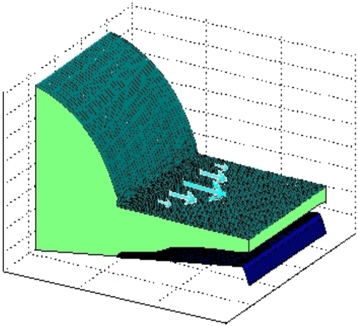



Figure: Van der Veen, Fundamental of Glacier Dynamics

# Buttressing/Stability Experiment

- On the other hand, rigid sidewalls lead to the presence of horizontal shear
- We expect that the narrower the channel, the more the walls are "felt", and maybe instability can be reversed



# Buttressing/Stability Experiment

- In this experiment, simulations were done on a foredeepened bed with rigid sidewalls
- Constant Parameters:
  $L$ (length): 1,500 $km$
  $a$ (accumulation): 0.3 $m/a$ (no melting)
  $A$ (Glen's Law constant): equivalent to $-20^\circ$ C
  Depth at shelf front: 600 $m$
- Varied Parameters:
  $\alpha$ (bedrock slope): between .001 and .00025
  $W$ (channel width): 150-1000 $km$
  $C$ (friction coefficient): 5-15 MPa$(s/m)^{\frac{1}{3}}$

# Buttressing Experiment

- The goal: for each ($W$,$\alpha$) pair, how long (if ever) until **VAF** (volume above flotation)$\rightarrow$ 0? *e.g*:

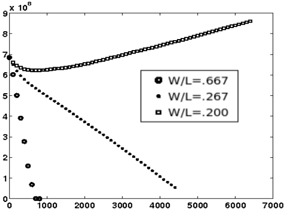
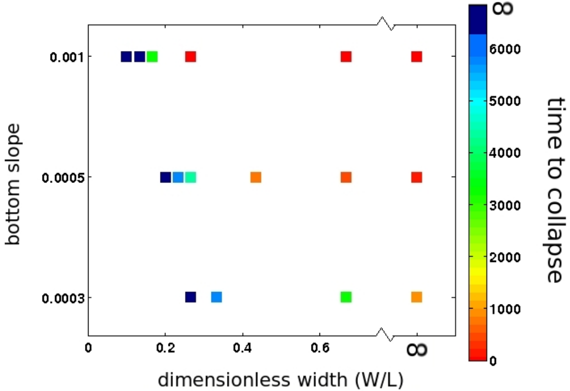



Figure: VAF vs time, $\alpha = 5 \times 10^{-4}$

- Our criteria for collapse is $VAF = 0.1 \times VAF_0$

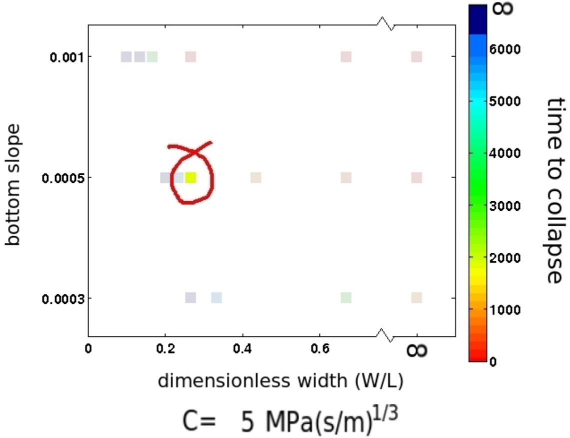
# Buttressing Experiment Results - Base Case




$C = 7.6\,\text{MPa}(\text{s/m})^{1/3}$

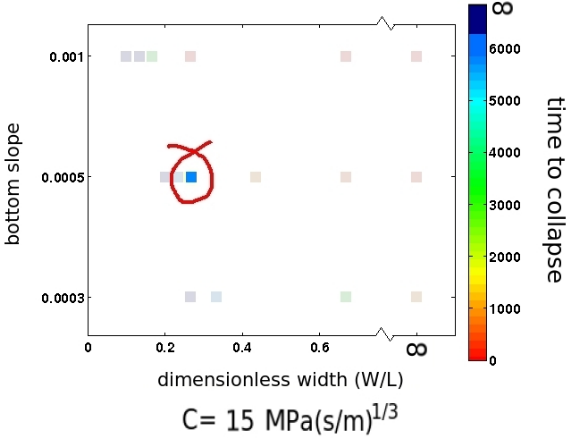
**Color = years to collapse** (i.e. when $VAF = 0.1 \times VAF_0$)

# Buttressing Experiment Results - Low $C$



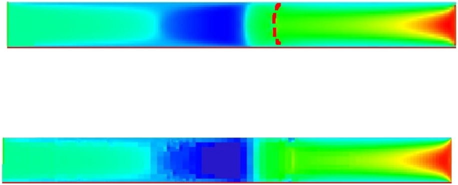

We see that decreasing basal strength can quicken collapse by several millenia..

# Buttressing Experiment Results - High $C$



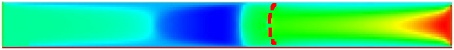

And we see that increasing basal strength can slow collapse by a comparable amount.

# Is Adaptive Refinement ever good?



- For a narrow channel ($W/L = 0.1$) we look at along-flow longitudinal stress ($\sigma_{xx}$). Without high tensile stress at the grounding line (red line), there is no need for ultra-fine resolution there.
- A highly-refined uniform mesh (top) agrees well with an Adaptively Refined mesh (bottom).

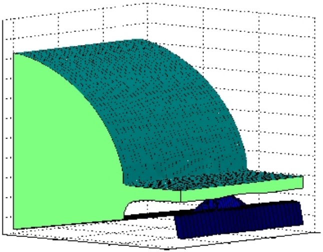
# Is Adaptive Refinement ever good?



- However, we see that a Moving Mesh does a pretty bad job of representing $\sigma_{xx}$.
- This may be because of distortion of the Moving Mesh where the grounding line intersects the side boundaries. If this is the case, then a moving mesh may not only be unnecessary with strong buttressing but counterproductive.

# Application: Ice Rises (& movies)

- One way to view an ice rise is that it effectively reduces the width of an ice shelf
- Does it change stability? Does it agree with the previous experiment?
- Experiment: Channel 550 *km* wide, bed as before but with rise $\sim$ 200m below S.L. - does it prevent collapse?



# Conclusions

- The ability to successfully simulate grounding line migration in a 2D context will be valuable to more realistic (less idealized) models of marine ice sheets
- Buttressing by sidewalls (and even more so by ice rises) is shown to be sufficient to reverse or dampen marine ice sheet instability at realistic scales; the effect is also sensitive to basal strength
- In many of the cases we examined, Moving Mesh seems to be stronger than Adaptive Refinement. However, we have seen it fall down..

# Further Work

- Inclusion of effects which can be expected to *systematically* affect results (calving of long ice shelves, weakening of ice in shear margins, bedrock adjustment)
- Mass balance was uniform - how would a realistic ocean change the melt pattern in response to ice shelf/grounding line evolution?
- Despite its weaknesses, Adaptive Refinement has the promise to be more versatile, and more amenable to coupling with ocean or realistic bedrock